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AN INVESTIGATION INTO THE VALUE OF THE

MULTIPLE FORCE EMPLOYMENT VARIATION

FLEXIBILITY OPTION

AFIT/GOR/OS/81D-7

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AN INVESTIGATION INTO THE VALUE OF THE MULTIPLE FORCE EMPLOYMENT VARIATION FLEXIBILITY OPTION

THESIS

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by

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Contents

List of Figures

Figure		Page

List of Tables

Abstract

The change in the destruction capability of a bomber force resulting from the incorporation of the Multiple Force Employment Variation (MFEV) was estimated under a variety of conditions. The study varied the distribution of target values, the bomber's base escape survival probability, the number of preplanned options, and targeting philosophy. The resulting value of targets destroyed under the MFEV were compared with the value of targets destroyed under a single-plan system, under identical conditions.

Targeting philosophy was found to be a major determinant of MFEV value. The value when all targets must be assigned in every plan was compared with the value with that requirement relaxed. The restriction was found to be very important in estimating MFEV value, particularly for survival probabilities lower than .6. Without the restriction in effect, about two-thirds of the value of retargeting was achieved by the MFEV. The value of retargeting is defined as the difference between full flexibility and a single-plan case as is now employed. When all targets had to be assigned in each plan, only about one-third of the value of retargeting was realized.

The value of retargeting was found to be very sensitive to the aircraft probability of surviving an enemy surprise attack on bomber bases. For low survival probabilities, the capability of retargeting was found to be very important. As the survival probability increased, however, the value of the MFEV decreased.

The distribution of target values also was found to be an important determinant of MFEV value. Target values sampled from an exponential distribution resulted in the highest percentage MFEV value. The lowest

values occurred when the target values followed a normal distribution with a strong central tendency. Ranked (integer) target values and values following a uniform distribution resulted in a measure about midway between the two extremes.

The number of plans above two was not found to be a strong determinant of MFEV value. Any increase in the number of plans, however, was found to increase the value of the system.

I Statement of the Problem

Introduction

Presently, if an incoming attack were to scramble the United States manned bomber force, no retargeting would be possible. Once the aircraft were airborne, there would be no way to change the target set assigned to each aircraft. Even if it were known, for example, that some targets were no longer of value to the enemy (perhaps destroyed by missiles shortly after the aircraft were airborne), nothing could be done to redirect the bombers ordered against these targets. In addition, if some bombers were destroyed by the enemy attack, failed to rendezvous with a tanker for refueling, or had to abort their mission for some other reason, their targets would not be attacked since airborne retargeting is not presently possible.

Statement of the Problem

Under the present targeting system, in the event of an enemy first strike, many of the high value enemy targets would never be attacked because the weapons targeted against them would be destroyed in the initial attack. This deficiency in the capability of the United States strategic forces reduces their effectiveness as a deterrent.

Background

At least three strategies could be used to increase the deterrent capability of the U.S. forces. First, massive numbers of redundantly-targeted weapons could be constructed. Then, in the event of an enemy strike, enough weapon systems would survive to attack most enemy high-value targets. Second, a defensive net could be constructed to protect the strategic offensive forces from an enemy first strike, thereby

ensuring enough forces would survive to attack the "most important" targets. Third, the U.S. targeting system could be adapted so that those weapon systems which do survive an enemy surprise attack could be retargeted against the "most valuable" targets. This thesis focuses on one approach to the third strategy -- airborne retargeting.

Many adaptations to incorporate the retargeting into the Single Integrated Operations Plan (SIOP) could be conceived. (The SIOP is the plan which assigns strategic forces to their duties in time of war.) Since the manned bomber once airborne is inherently more flexible than ballistic missiles, any inflight retargeting scheme would pertain mainly to penetrating bombers or their cruise missile payloads. The Multiple Force Employment Variation (MFEV) is one such retargeting plan.

Consider a modification to the present assignment procedure. Rather than a single SIOP, a set of plans are formulated, each plan allocating aircraft to target sets differently. After an enemy attack, when survival information was available, the commander could select the best plan. He could analyze which aircraft remained operational and which enemy targets retained their value through the U.S. missile attack. He could then select that plan which would maximize the expected value of the enemy targets to be destroyed under those specific conditions. This flexibility option is known as the Multiple Force Employment Variation. The idea of the MFEV could also be applied to the other legs of the strategic triad. Multiple plans could be created in advance, allocating missiles to targets differently in each plan, similar to the procedure suggested for a bomber force. After an enemy attack, the commander could select that plan which had the best assignment of missiles to targets, for the specific missiles that survived the enemy attack.

Thus, the MFEV could also be used to increase the effectiveness of a surviving second-strike missile force.

A flexibility option such as MFEV would increase the total value of enemy targets destroyed in the event of a surprise attack against the United States, thereby increasing the effectiveness of the U.S. bomber force as a deterrent. But by how much is the deterrence increased? Is the expected increase in the value of enemy targets killed worth the costs, both hardware and software, of designing, developing and implementing such a system? Could the money required for converting the bombers and command and control systems be better spent on some other weapon system? These questions can only be answered if the benefits of such a system are known or have been estimated.

Objectives

The primary objective of this research is to obtain an estimate of the value of the MFEV as compared with a system without airborne retargeting. The cost and feasibility issues will not be considered.

Scope

Research into the value of the MFEV could follow at least three separate paths, as illustrated in Figure 1. The three axes shown represent the three directions. The inner, solid box represents the work done by Dimon (Ref 3). The dashed lines enclose the region researched by this thesis. The specific topic areas shown in this figure will be discussed in the Recommendations section of Chapter VII.

The value of the MFEV is thought to be dependent upon a number of factors:

1) differences in the probability distribution of target values,

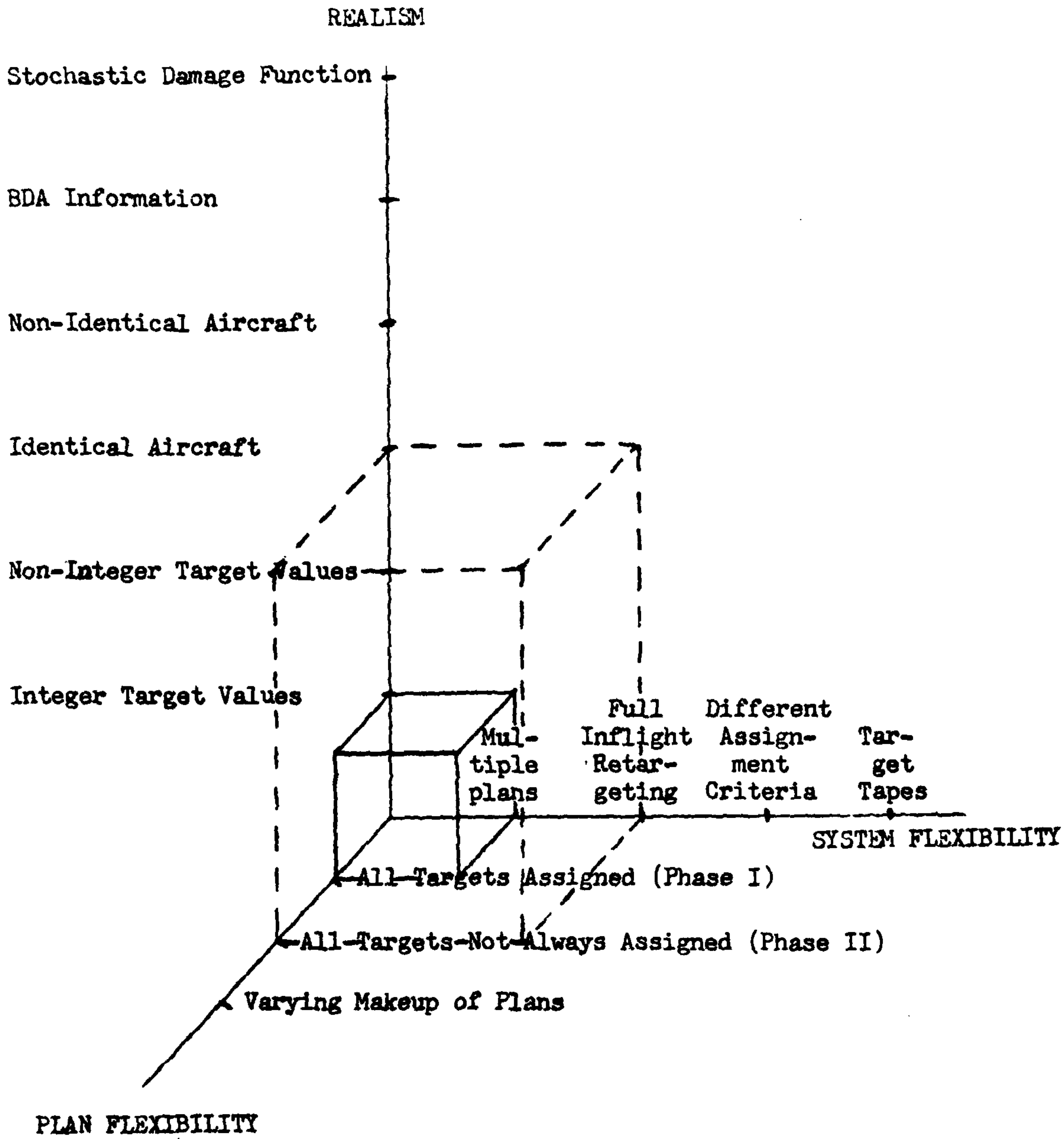

Figure 1. MFEV Research Areas

2) differing values of aircraft survival probability,

3) target selection criteria, and

4) the number of preplanned MFEV options.

The above factors will be varied and the MFEV values obtained under differing assumptions will be compared to determine the sensitivity of the value to each of them.

All aircraft are assumed homogeneous and have an equal probability of surviving the enemy attack. Any aircraft which survives, kills its target with probability one. That is, the conditional probability that an aircraft destroys its intended target, given that the plane survived the enemy attack, is one.

Approach

The model begins with a given set of twenty aircraft and twenty target sets. Each target set may consist of multiple targets, but the set itself is considered as a single unit and evaluated as such. Thus, in this thesis, the terms target and target set are used interchangeably.

Twenty samples are taken from each of five distributions to determine target values. The identical methodology is then followed for each set of targets.

First, the assignment procedure is accomplished. This process corresponds to the building of a multi-plan SIOP. The assignment of aircraft to targets in multiple, complementary plans is not a trivial task. The SIOP is essentially a constrained, two-dimensional assignment problem, allocating aircraft to targets. The two-dimensional assignment problem is considered a simple problem, as the difficulty of problems is measured (Ref 4:50-53), yet thousands of man and computer hours are needed each year to complete the SIOP. The allocation of aircraft to targets

in the MFEV, on the other hand, is thought to be a "hard" problem. The MFEV is closely related to a three-dimensional assignment problem (allocating aircraft to targets and plans), and as such, is thought to be inherently intractable; that is, the three-dimensional assignment problem is a member of a class of over 300 hard problems for which no polynomial time algorithm has yet been found to solve any of them optimally (Ref 4:8, 50-53). Therefore, rather than exert an extensive amount of time searching for an optimal solution, a heuristic technique is developed and used to find a near optimal assignment. A heuristic is a solution technique which, although not guaranteeing optimality, can obtain "good" solutions to large problems with limited computational effort (Ref 1:163).

Second, the enemy attack is simulated and surviving aircraft determined. For these surviving aircraft, statistics are collected on the value of the MFEV versus the value of the single-plan assignment system, similar to the current SIOP. One thousand enemy attacks are simulated.

The results of the simulation are analyzed by comparison with other targeting strategies. The value of the MFEV is compared with a single-plan assignment system and with a retargeting system which has the highest possible flexibility -- the system which is able to attack the j most important targets when only j aircraft survive the enemy strike.

Sequence of Presentation

Chapter II presents the theoretical basis for the results used throughout this thesis. The difficulty of the MFEV assignment problem is discussed and different single-plan assignment strategies proved optimal under differing assumptions. The statistics to be collected

in this research are derived.

The third chapter serves as the framework which holds the research together. Here the overall design is discussed, together with specifics on procedures and assumptions.

The fourth and fifth chapters explain the computerization of the model. Chapter IV explains the Phase I version -- all aircraft allocated against distinct targets. Changes made to the system to allow multiple aircraft to be assigned against the same targets are explained in Chapter V.

Chapter VI presents and analyzes the data accumulated from the simulation model. The results are viewed from several directions, and compared with different measures. Conclusions are drawn about the value of the MFEV and the sensitivity of the value measurement to changes in the factor levels.

The seventh chapter summarizes the conclusions made in Chapter VI. Also, avenues for future research are recommended.

II Theoretical Development

This chapter develops the theoretical base for the analysis documented in subsequent chapters. First the difficulty of the MFEV assignment problem is discussed with emphasis on the obtainability of an optimal solution in a reasonable amount of time. Then a variety of general target assignment methodologies are presented and reference made to the use that was made of the result. Finally, the measures of merit utilized in this research are explained. A description of the computer code based on these results can be found in Chapters IV and V.

Mathematical Model

As mentioned in Chapter I, the allocation of aircraft to targets in a MFEV system is a three-dimensional assignment problem with a non-linear objective function. A formulation for the case when all aircraft must be assigned to distinct targets is as follows:

Maximize: the expected value of targets destroyed

$$\text{Subject to:} \quad \sum_{j=1}^{n} X_{ijk} = 1 \qquad \text{for } i=1, \ldots, n; \quad k=1, \ldots, m \qquad (1)$$

$$\sum_{i=1}^{n} X_{ijk} = 1 \qquad \text{for } j=1, \ldots, n; \quad k=1, \ldots, m$$

where $X_{ijk} = 1$ if aircraft i is assigned to target j in plan k

$= 0$ otherwise

It should be noted that the three-dimensional assignment problem is usually formulated with a third set of constraints, summed over the third index. Since these constraints were excluded, an aircraft could

conceivably be allocated against the same target in more than one plan.

The objective function is easily described in words but is not so easily described mathematically. The basic difficulty lies in the fact that the goal "to maximize the expected value of targets destroyed" is actually the largest order statistic from a set of dependent random variables -- the values of the m plans. The plan of largest value for a given state of nature is the plan selected. So the system always takes on the value of its largest element. The value of more than one plan may be "large" for a given state of nature, but only one plan may be selected. Therefore, having two plans of "large" value is wasteful in that the value of both plans under other states of nature must be correspondingly "small" (Ref 3:C23). Since no optimal formulation of this objective function was discovered, a heuristic iterative improvement procedure based on criteria suggested by Dimon was used to determine a near-optimal solution (Ref 3).

Heuristic solution techniques are used to obtain solutions to large problems with only limited computational effort. Normally, they are used in cases of diminishing marginal returns of computer resource -- they determine near-optimal solutions with much less computer effort than would be needed to determine the optimal solution (Ref 1:163). The heuristic procedure used in this research is described in Chapters II and IV.

NP-Hardness

It appears that the MFEV assignment problem, like the general three-dimensional assignment problem, is NP-Hard (Ref 4:50-53). Although a discussion of the class of NP-Hard problems lies beyond the scope of this paper, a consequence of NP-Hardness drastically impacts the optimal

solvability of this class of problems. No algorithm has yet been found which can solve, in a reasonable amount of computer time, any but the most trivial NP-Hard problem, (the solution time is normally an exponential function of the size of the problem) (Ref 4). For this reason, a heuristic solution technique was devised to find a near-optimal MFEV assignment.

Optimal Single-Plan Assignment

In this section, two different single-plan assignment methodologies are proven optimal under differing criteria. First, the optimal allocation when no two aircraft may be assigned against the same target will be demonstrated. Second, the optimal assignment will be developed for the case when the target assignment need not be distinctive.

Distinct Targets. In this research, the value of any system is measured relative to the base case -- the single plan assignment which allocated aircraft to targets in order to maximize the expected value of targets destroyed while assigning exactly one aircraft against each target. The following lemma describes the optimal assignment under these conditions.

Lemma 1 -- Suppose the aircraft are ordered by non-increasing probability of survival (i.e. $P_1 \geq P_s \geq \ldots \geq P_n$ where P_i = probability of survival of aircraft i for $i = 1, 2, \ldots, n$). Also suppose the targets are numbered such that $V_1 \geq V_2 \geq \ldots \geq V_n$; where V_j = value of target j for $j = 1, 2, \ldots, n$; and n is the number of aircraft and targets. Then the assignment which maximizes the expected value of targets destroyed associates aircraft i to target i, for $i = 1, 2, \ldots, n$.

Aircraft 1, 2, ..., i, ..., k, ..., n

Target 1, 2, ..., j, ..., i, ..., 1

Figure 2. Initial Assumed Assignment (I^1)

Aircraft 1, 2, ..., i, ..., k, ..., n

Target 1, 2, ..., i, ..., j, ..., 1

Figure 3. Assignment After a Single Interchange (I^2)

<u>Proof</u> -- Let the aircraft and targets be ordered as assumed in the lemma ($P_1 \geq P_2 \geq \ldots \geq P_n$ and $V_1 \geq V_2 \geq \ldots \geq V_n$). Let I be the assignment suggested in the lemma and let I^1 be some other assignment of aircraft to targets such that I and I^1 differ. Let i be the first position in I^1 such that I and I^1 differ. Let j be the target assigned to plane i in I^1 and k be the plane which is assigned against target i in I^1. (See Figure 1 for an illustration). Now interchange targets i and j in I^1 to form a new assignment I^2 as shown in Figure 2.

The objective function values for I^1 and I^2 differ only in the i^{th} and k^{th} positions and are given by

$$EV(I^1) = P_1V_1 + P_2V_2 + \ldots + P_iV_j + \ldots + P_kV_i + \ldots + P_nV_1 \tag{2}$$

$$EV(I^2) = P_1V_1 + P_2V_2 + \ldots + P_iV_i + \ldots + P_kV_j + \ldots + P_nV_1 \tag{3}$$

Subtracting Eq (2) from Eq (3)

$$\begin{aligned} EV(I^2) - EV(I^1) &= P_iV_i + P_kV_j - (P_iV_j + P_kV_i) \\ &= P_i(V_i - V_j) + P_k(V_j - V_i) \\ &= (P_i - P_k)(V_i - V_j) \\ &\geq 0 \end{aligned} \tag{4}$$

since $P_i \geq P_k$, $V_i \geq V_j$ by definition

Hence, the interchange cannot produce an inferior solution and may produce a superior one. After at most $n - 1$ of these interchanges, the resulting solution is identical to I. Hence, the assignment given by Lemma 1 is optimal and the Lemma is accepted.

Distinct Targets Not Required. Phase I of this research required that each aircraft be assigned against a unique target in a given plan. It may well be, though, that the best targeting strategy assigns more than one aircraft against some of the targets and consequently, none at all against some others. Phase II explores this approach and allows multiple aircraft to be allocated against the same target in any plan.

The targeting logic utilized in Phase II was to maximize the marginal value of each aircraft as it was assigned. Thus, each aircraft, in turn, is allocated against the target where it would be expected to do the most good -- where the marginal value is highest. Once the single-plan assignment is determined, the number of aircraft assigned against each of the targets in that plan determines the number of aircraft assigned against each target in the other MFEV plans. Thus, the number of aircraft assigned to each target is identical under all plans. The following lemma describes the optimal solution for the single-plan assignment when targets need not be distinct.

Lemma 2 -- Let V_i be the value of target i, P_s be the probability that a given friendly aircraft survives an enemy attack, and P_{k_j} be the probability that an attacking aircraft kills target j. The optimal assignment of planes to targets allocates each aircraft, in turn, against the target with the largest remaining marginal value. The marginal value of target i (MV_i) represents the value of assigning one more aircraft against target i when j aircraft have already been assigned

against it. It can be expressed mathematically by the following expression:

$$MV_i = P_s P_{k_i} (1 - P_s P_{k_i})^j V_i \tag{5}$$

<u>Proof</u> -- This proof has two parts. First, the above expression will be shown to represent the marginal target value. Secondly, it will be shown that assigning aircraft to targets by means of remaining maximum marginal value does maximize the expected value of targets destroyed, and thus is the optimal assignment for that objective.

The probability that aircraft m kills target i is just the intersection of the two events A and B, where A is the event that aircraft m survives the initial enemy attack and B is the event that an attacking aircraft successfully destroys target i, and A and B are assumed independent. The probability of the intersection of two independent events is just the product of their probabilities (Ref 6:42), so:

$$\begin{aligned} P(\text{aircraft m kills target i}) &= P(A)\, P(B) \\ &= P_{s_m} P_{k_i} \end{aligned}$$

where P_{s_m} = probability that aircraft m survives the initial attack

P_{k_i} = probability that an attacking aircraft kills target i -- conditional probability of a kill given the aircraft survives to attack

The probability that the target survives the aircraft's attack is just the probability of the union of the complements or one minus the above product, as shown below (Ref 6:15):

$$\begin{aligned} P(\bar{A} \cup \bar{B}) &= P(\bar{A}) + P(\bar{B}) - P(\bar{A} \cap \bar{B}) \\ &= (1 - P_{s_m}) + (1 - P_{k_i}) - (1 - P_{s_m})(1 - P_{k_i}) \\ &= 1 - P_{s_m} P_{k_i} \end{aligned} \tag{7}$$

The probability that target i survives two such aircraft targeted against it is just the product of two terms of the form given above (assuming independence between aircraft)(Ref 6:24).

In this research, all aircraft are assumed to have an equal probability of survival. Using this assumption ($P_{s_i} = P_s$, for all i), the probability that target i survives when j aircraft are assigned against it is given by the following expression:

P(target i survives when targeted by j identical aircraft)

$$= \prod_{m=1}^{j} (1 - P_s P_{k_i})$$

$$= (1 - P_s P_{k_i})^j \tag{8}$$

This same probability is also the expected proportion of target value remaining after j aircraft have been assigned against target i. Thus, a residual value of $(1 - P_s P_{k_i})^j V_i$ remains out of the initial target value of V_i.

The assignment objective is to allocate each aircraft against the target where it would do the most good. The marginal value of target i (MV_i) is the value of assigning one more aircraft against target i when j aircraft have already been assigned against it. Thus, the best target to allocate an aircraft against is the one with the highest remaining marginal value. The marginal value equals the difference between the residual value when j aircraft have been assigned against target i and the residual value if $j + 1$ aircraft were assigned. Thus,

$$MV_i = (1 - P_s P_{k_i})^j V_i - (1 - P_s P_{k_i})^{j+1} V_i$$

$$= (1 - P_s P_{k_i})^j (1 - (1 - P_s P_{k_i})) V_i$$

$$= P_s P_{k_i} (1 - P_s P_{k_i})^j V_i \quad (9)$$

This expression will be referenced in Chapter V and was used in the computer algorithm to determine how many aircraft to assign against each target.

Now, it remains to show that this is the optimal assignment. Our objective is to maximize the expected value of targets destroyed. The expected value of targets destroyed is the sum of the expected value contributed by each aircraft because the aircraft are assumed independent. Since at each decision point, the marginal value was selected so that it was as large as possible, the sum must be as large as possible. Thus, if one wa. to replace any assignment with one that has a smaller marginal value, the sum (expected value) would be reduced. Therefore, assignment by largest marginal value is the optimal allocation scheme. This completes the proof of Lemma 2.

Post-Simulation Value Measurement

Once the assignment procedure has been completed, the states of nature are simulated by a Monte-Carlo simulation, as is discussed in Chapter III. These states of nature represent the specific aircraft which survive a simulated enemy attack, given a common probability of survival for each aircraft. The expected value of targets destroyed, given perfect knowledge of the aircraft that survive, can then be measured as the sum of the expected value of targets destroyed by each of the surviving aircraft.

When all aircraft are assigned to distinct targets, the expected value contributed by each aircraft is independent of all other aircraft. Therefore, the value of each plan is given by the following equation:

$$E(\text{value targets destroyed}) = \sum_{i=1}^{n} X_i P_{k_i} V_i \qquad (10)$$

where P_{k_i} = Probability that an attacking aircraft kills target i

V_i = Value of target i

X_i = 1 if the aircraft assigned against target i survives
= 0 otherwise

The measurement of the value of a plan is more complicated when the distinct target constraint is relaxed. Under the Phase II case, multiple surviving aircraft may be assigned against the same target. One must insure that the same target is not counted as destroyed more than once. Obviously, the value contributed by each target depends on the number of planes assigned against it. The targets were evaluated at their current marginal value as the contribution from each aircraft was determined, in turn. The rationale for this is actually just a special case of the previously derived expression for the marginal value of a target, Eq (9), when specific aircraft are known to survive with probability one ($P_s = 1.0$). The definition of j is slightly altered, however. The expression shown below was used to evaluate the value of each plan in Phase II of this research.

$$E(\text{Value of targets destroyed}) = \sum_{i=1}^{n} X_i \sum_{j=1}^{m_i} (P_{k_i} (1 - P_{k_i})^{j-1} V_i) \qquad (11)$$

where n = number of targets

m_i = the number of surviving aircraft assigned against target i in this plan

P_{k_i} = probability that an attacking aircraft kills target i

V_i = value of target i

i = index of the target array

$X_i = 1$ if $m_i \geq 1$
$= 0$ otherwise

Obviously, if no aircraft assigned to target j survive, target j contributes nothing to the value of the plan. As an example, let $m_i = 3$ for some i (three of the surviving aircraft are targeted against target i in the plan being measured). Then the contribution of this target to the expected value is given by the following term:

$$\begin{aligned}
\text{Contribution by target } i &= \sum_{j=1}^{3} \left(P_{k_i}(1 - P_{k_i})^{j-1}\, V_i\right) \\
&= P_{k_i}(1 - P_{k_i})^0\, V_i + P_{k_i}(1 - P_{k_i})\, V_i \\
&\quad + P_{k_i}(1 - P_{k_i})^2\, V_i \\
&= P_{k_i}(1 + (1 - P_{k_i}) + (1 - P_{k_i})^2)\, V_i \\
&= P_{k_i}(3 - 3P_{k_i} + P_{k_i}^2)\, V_i \qquad (12)
\end{aligned}$$

The distinct targets evaluation scheme mentioned earlier is really just a special case of this marginal value measure (with $m_i = 1$ for all aircraft). This can be seen by the following adaptation of Eq(11):

E(value of targets destroyed, Phase I)

$$\begin{aligned}
&= \sum_{i=1}^{n} X_i \sum_{j=1}^{1} (1 - P_{k_i})^{j-1}\, V_i) \\
&= \sum_{j=1}^{n} X_i P_{k_i} (1 - P_{k_i})^0\, V_i \\
&= \sum_{i=1}^{n} X_i P_{k_i} V_i \qquad (13)
\end{aligned}$$

where all variables are consistent with their previous definitions

Derivation of Statistics Collected

Three measurements were created to compare the value of the MFEV with different figures. The percent improvement over the base case compares the MFEV against a single-plan assignment by a ratio of their values. The percent total value demonstrates the maximum expected value that any assignment could possibly approach, given an aircraft probability of survival. The percent of the best possible solution compares the value of the MFEV with the value of the optimal assignment (best possible solution) for given aircraft survival probabilities, where the optimal assignment is defined as the allocation which attacks the j highest-valued targets when exactly j aircraft survive.

Percent Improvement Over the Base Case. The basic measure of merit throughout this research is the percent improvement over the base case (P), a ratio of two values, each averaged over one thousand trials. Data generated from this procedure drive most of the other measurement procedures.

The following expression describes the data which was gathered:

$$\begin{aligned} P &= 100\left(\frac{\overline{V}}{\overline{B}} - 1\right) \\ &= 100\left(\frac{\Sigma V_i}{\Sigma B_i} - 1\right) \\ &= 100\left(\frac{\Sigma V_i - \Sigma B_i}{\Sigma B_i}\right) \\ &= 100\left(\frac{\Sigma (V_i - B_i)}{\Sigma B_i}\right) \end{aligned} \tag{14}$$

where V_i = value of the best plan, trial i

B_i = value of the base plan, trial i

$\overline{V}$ = average V_i over the 1000 trials

$\overline{B}$ = average B_i over the 1000 trials

Note: all sums are taken over all 1000 trials.

For a given trial i, $V_i \geq B_i$, so the ratio of the averages has a lower bound of one, which in turn implies $P \geq 0$, since one was subtracted to shift this lower bound to zero. Multiplication by one hundred was used to scale the value to percent.

The measure, P, mentioned above is the "ratio of the averages." A similar estimator, the "average of the ratios" (Q), was considered as an alternative measure of merit. This estimator is shown in the following equation:

$$Q = 100\,\Sigma\left(\frac{V_i}{B_i} - 1\right) / 1000 \qquad (15)$$

Q has some unpleasant properties. First, it has a higher variance than P since small deviations from the mean of V and from the mean of B can interact to cause rather large deviations from the expected value of B/V. Second, if V and B are assumed to be independent, normal random variables, Q is related to a Cauchy distribution, which doesn't have an expected value. Also, the sample mean of random draws from a Cauchy distribution does not converge as the number of samples increases -- one sample is just as likely to be close to the true value of the ratio as is the sample mean over 1000 trials (Ref 2:421).

P, on the other hand, has some valuable properties. Since $\overline{V}$ is the best estimate for the expected value of V (the value of the best plan) and $\overline{B}$ is the best estimate for the expected value of B (the value of a single plan), $\overline{B}/\overline{V}$ should be a good estimate for the true ratio of B to V (Ref 10). For these reasons, P was selected as the measure of merit for this research.

<u>Percent Total Value</u>. The percent total value measurement (T) shown in Chapter VI is the value of the most efficient bomber attack,

measured in percent of the total target value, that would be expected for a given P_s. If all aircraft are expected to survive an attack ($P_s = 1$), $T = 100$. For all other P_s values, the value is less. The measure does not exist at $P_s = 0$.

The values for T are calculated at each of five levels of P_s for each distribution in the following manner:

Step 1: For each level of P_s (.2, .4, .6, .8, 1.), determine the expected number (N) of aircraft surviving out of twenty. That is, $N = 4, 8, 12, 16, 20$, respectively.

Step 2: Set T equal to the average sum of the N largest target values taken over all target sets drawn from a specific distribution, multiplied by one hundred and divided by the sum of all twenty target values.

For any distribution, T is a strictly increasing function of P_s such that any point (P_s, T) lies on or above the line connecting (1, 0) and (1, 100). T would be linear only if all of the targets in the set had the same value. For any other distribution of target values, the slope of the curve depends on P_s. For large values of P_s, only the smallest targets fail to be counted. As P_s decreases through the medium-size range, none of the smaller targets are included and only the largest of the medium-size ones. For each incremental decrease in P_s for P_s small, a larger and larger value target is not included in the percent of total value sum.

Thomas produced an expression for the percent of total value for ranked values consisting of targets of value equal to the first n positive integers (identical to the integer "distribution" used in this

research) (Ref 9). Thus, the most important target has value n and the least valuable target is worth one unit. The general expression for an arbitrary number of aircraft is derived below. Specific expressions for twenty and an infinite number are also given and graphed in Figure 40, together with the linear expression (all target values equal). The horizontal scale was reversed from Thomas' work to remain consistent in this paper.

$$D = \frac{\sum_{i=1}^{d} (n + 1 - i)}{\sum_{i=1}^{n} i}$$

$$= \frac{\sum_{i=1}^{d} (n + 1) - \sum_{i=1}^{d} i}{\sum_{i=1}^{n} i}$$

$$= \frac{d(n + 1) - \frac{d(d + 1)}{2}}{\frac{n(n + 1)}{2}}$$

$$= \frac{d(2n - d + 1)}{n(n + 1)} \qquad (16)$$

but $E(d) = nP_s$, so $E(D) = \dfrac{nP_s(2n - nP_s + 1)}{n(n + 1)}$

$$= \frac{P_s(n(2 - P_s) + 1)}{n + 1} \qquad (17)$$

where n = number of targets

d = number of targets attacked

Substituting $n = 20$ into Eq(17) gives the following result:

$$E(D) = \frac{41P_s - 20P_s^2}{21} \qquad (18)$$

The limiting case, as the number of targets grows infinitely large, is given by the following expression:

$$E(D) = (2 - P_s)P_s \tag{19}$$

<u>Percent of the Best Possible Solution</u>. The percent of the best possible solution measurement is derived from the two previously mentioned measures (percent improvement over the base case and percent total value). The value of the MFEV obtained through simulation is divided by the expected value of the best possible assignment. The value of the best possible assignment for a given set of targets is defined as the sum of the j largest target values when exactly j aircraft survive the initial enemy attack. This value is estimated for each distribution by the following procedure:

Step 1: Calculate the expected value of a single-plan, discrete target assignment (E(B)) by the following expression:

$$E(B) = \frac{P_s \sum_{i=1}^{10} \sum_{j=1}^{n} V_{ij}}{10} \tag{20}$$

where V_{ij} = j^{th} largest target in target set i

n = initial number of aircraft

In other words, calculate the average total value of a target set, averaged over the ten sets sampled from each distribution.

Step 2: For each of the four levels of P_s (.2, .4, .6, .8), determine the expected number of surviving aircraft (N)

$$N = nP_s \tag{21}$$

Step 3: Calculate the sum (S) of the largest N target values for each level of P_s (averaged over the ten sampled sets of targets). This is labelled the value of the best possible solution.

$$S = \frac{\sum_{i=1}^{10} \sum_{j=1}^{N} V_{ij}}{10} \tag{22}$$

Step 4: Create the ratio (R) of the value of the MFEV to the value of the best possible assignment. Since percent improvement over the base case (P), derived in Eq(14), is the measure used to estimate the value of the MFEV, this figure must first be converted back to a value before the ratio can be taken. The conversion and division is done by the following expression:

$$\begin{aligned} R &= \frac{(P + 100)\ E(B)}{S} \\ &= \frac{\left(100\left(\frac{\bar{\bar{V}}}{\bar{\bar{B}}} - 1\right) + 100\right) E(B)}{S} \\ &= 100\,\frac{\bar{\bar{V}}}{S}\left(\frac{E(B)}{\bar{\bar{B}}}\right) \\ &= 100\,\frac{\bar{\bar{V}}}{S} \end{aligned} \tag{23}$$

Although $\bar{\bar{B}} \neq E(B)$ (simulated estimate not identical to analytically derived exact result), the two were verified as very nearly equal. The values were compared for this study and a difference of slightly over two percent was the largest deviation discovered. Most differences

were down in the one percent range. Therefore, although this mixing of sources may introduce some error into the estimate of R, the figure output from this algorithm should be close to the true value. Thus, R should be a reasonable approximation to the percentage of the best possible solution achieved by the MFEV.

The estimated percent best possible solution of the base case is used to compare with the MFEV values in Chapter VI. The measure is approximated by dividing the expected value of the base case by the estimated value of the best possible solution, which were derived earlier, as shown below:

$$EV = \frac{E(B)}{S} \tag{24}$$

where E(B) and S are defined in Eq (20) and Eq (22).

III Design of Experiment

The objective of this research is to measure the change in the destruction capability of a bomber force resulting from incorporating the Multiple Force Employment Variation (MFEV). Also of great interest is the sensitivity of this measure to changes in the distribution of target values and the probability of aircraft survival. A model was created to estimate the value of the MFEV at different levels of these, and other parameters. The research was separated into two phases. Phase I requires that all targets are assigned in each plan. Phase II relaxes this requirement and allows multiple aircraft to be allocated against a single target.

Initial investigation into this subject suggested the value of the MFEV could be highly dependent upon many factors. The research therefore had to be carefully designed to properly account for these variables. As will be discussed in the Methodology and Target Value Distribution sections, some factors were assumed constant, some included parametrically and some had to be allowed to vary randomly in the model.

Scenario

This research will center on a simulation of a bomber force of twenty aircraft and a target set list consisting of twenty targets with known values. Twenty aircraft and targets were selected as a trade-off between the desire for a large number of aircraft (to enhance the degree of operational validity) and a number small enough to keep the problem to workable proportions. Twenty was felt to be a reasonable trade-off between the two arguments.

Implicit Assumptions. Throughout this research, all aircraft will

be assumed homogeneous. Each aircraft has an equal probability of surviving the initial enemy attack on its airfield and an identical probability of killing a given target, dependent only upon the target being attacked. In addition, the survival probabilities for all aircraft and probability of any target kill by any aircraft given launch survival are assumed independent of one another.

The target values are assumed constant over time unless destroyed by a bomber. Any aircraft which survives the simulated enemy attack is assumed to destroy the target it attacks with probability one -- that is, the conditional probability that a specific target is destroyed, given that the aircraft assigned against it survives the enemy attack, equals one. In Phase II, when more than one surviving aircraft may be assigned against a single target in a plan, only one aircraft is credited with having destroyed the target -- i.e. no target is counted as killed more than once.

Measure of Merit

Since the aircraft are assumed homogeneous, any single-plan assignment which allocates a bomber against every target is equivalent, i.e. has an identical expected value of targets destroyed. Therefore, an arbitrary single-plan assignment was selected and labelled as the "base case." This assignment served as the control, against which the value of the MFEV under different conditions could be evaluated. Each time a state of nature was simulated, both the value of the system under the assumed conditions (V_i) and the value of this base case (B_i) was recorded. After 1000 states of nature (specific aircraft surviving the enemy attack) were simulated, a ratio of the average values was calculated to determine the average percent improvement in value over

the average value of the base case. Thus a reading of 0.0 translates into no improvement while 100.0 would mean twice as much value as the base case (100.0 percent improvement). Improvement of more than 100.0 percent is possible. This percent improvement over the base case statistic served as the basic measure of merit for this entire project.

This particular measure of merit seemed most appropriate for numerous reasons. First, it is a scaleless quantity. No matter on what scale the target values might be measured, percent improvement (additional value) would just have a multiplier effect -- measured in the same scale. Secondly, this measure would be more directly applicable to a variety of problems than any measure tied to a specific unit. In addition, it maintains significance at any parameter level where many other measures would not. For example, if a value-based measure of merit were in use, an expected system increase of ten units of value might be an extremely significant improvement if only a few aircraft were expected to survive an enemy attack. If, on the other hand, most of the bomber fleet were expected to survive, this ten units may be only a negligible improvement.

The only other research on this problem used two measures of effectiveness. The basic measure was the value of an assignment given that a particular number of aircraft were lost (comparable to a given value of P_s). The assignment which was optimal for every choice of number of surviving aircraft was defined as the optimal assignment. (Thus, in order to be considered optimal, the assignment also had to be suboptimal for every possible number of surviving aircraft and every possible combination of specific aircraft surviving.) This definition proved to be overly restrictive. Indeed, Dimon proved no such optimum exists for systems of six aircraft and three plans or larger (Ref 3:C31-41). The

other measure of merit was an adaptation of this basic measure and was utilized when no "optimal" assignment existed. It consisted of the percentage by which the expected value a particular solution fell short of the generally unreachable upper bound solution, for a given number of surviving aircraft. This upper bound solution is the best possible solution used in this research and described in Chapters II, III, VI, and VII. Also, the percent improvement over the base case weights each specific number of surviving aircraft by its probability of occurrence through the Monte Carlo simulation procedure mentioned later in this chapter.

Target Value Distributions

The other investigation into the value of the MFEV assumed target values equal to successive integers (Ref 3). This research expands the scope of that study as well as relaxes the assumption of consecutive integer-valued targets by treating the target values as random observations from a known probability distribution.

In order to test the robustness of MFEV effectiveness to changes in target value distribution, identical methodologies were followed to measure the value of the MFEV under a variety of distributions of target values. The values were selected by Monte-Carlo sampling, a method of producing random variates from the particular probability distribution being sampled (Ref 8:65). The distributions selected cover a wide range of possibilities and most lists of operational target sets could be considered to approximate one of these. The five distributions from which target values were sampled for use in this research are given in the following sections.

Successive Integers. In the integer target list, target values are

deterministically established as the first twenty successive integers. Thus, the target of highest importance has value 20.0 and the target of lowest importance is valued at 1.0 unit. This was the scheme utilized by Dimon (Ref 3).

Uniform. Under the uniform target distribution, target values were randomly selected from a uniform distribution between 0.0 and 100.0. Since all values in the range have equal probability of occurrence, one would expect the most highly regarded target to be valued in the 90's and the target of least importance to be worth between 0.0 and 10.0 units. A sample graph of a uniform probability distribution function (pdf) is shown in Figure 4.

Low-Variance Normal. Target values for the low-variance normal distribution were sampled from a truncated (0, 100) normal distribution with mean 50.0 and a standard deviation of 10.0. This distribution would be expected to result in a large number of medium-valued targets and only a very few target values greater than 20.0 units away from the mean. A graphical example of this pdf is shown in Figure 5.

High-Variance Normal. Under the high-variance normal target list, target values were selected from a truncated (0, 100) normal distribution with mean 50 and standard deviation 30. Thus, in sampling from this distribution, one would expect a loose cluster around the mean with a good percentage of the values more than 20.0 units away from the mean. An example of a truncated high-variance normal pdf is given in Figure 6.

Exponential. Target values for the exponential target list were obtained from a truncated ($\leq$ 100) exponential distribution with mean 25. (The expected sample mean is actually about 23.134 due to the truncation at 100). This distribution approximates the case of a great

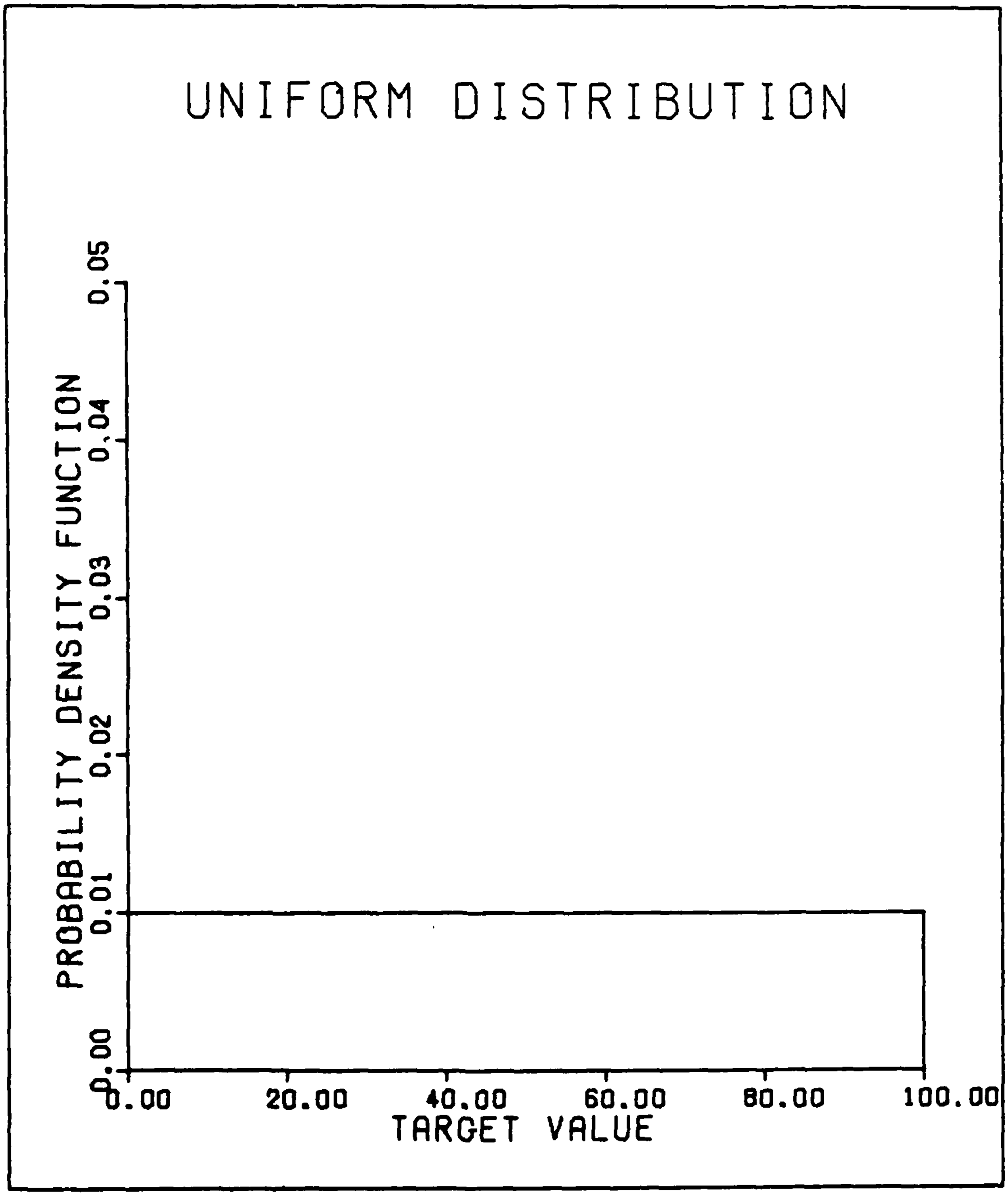

Figure 4. Uniform Probability Density Function

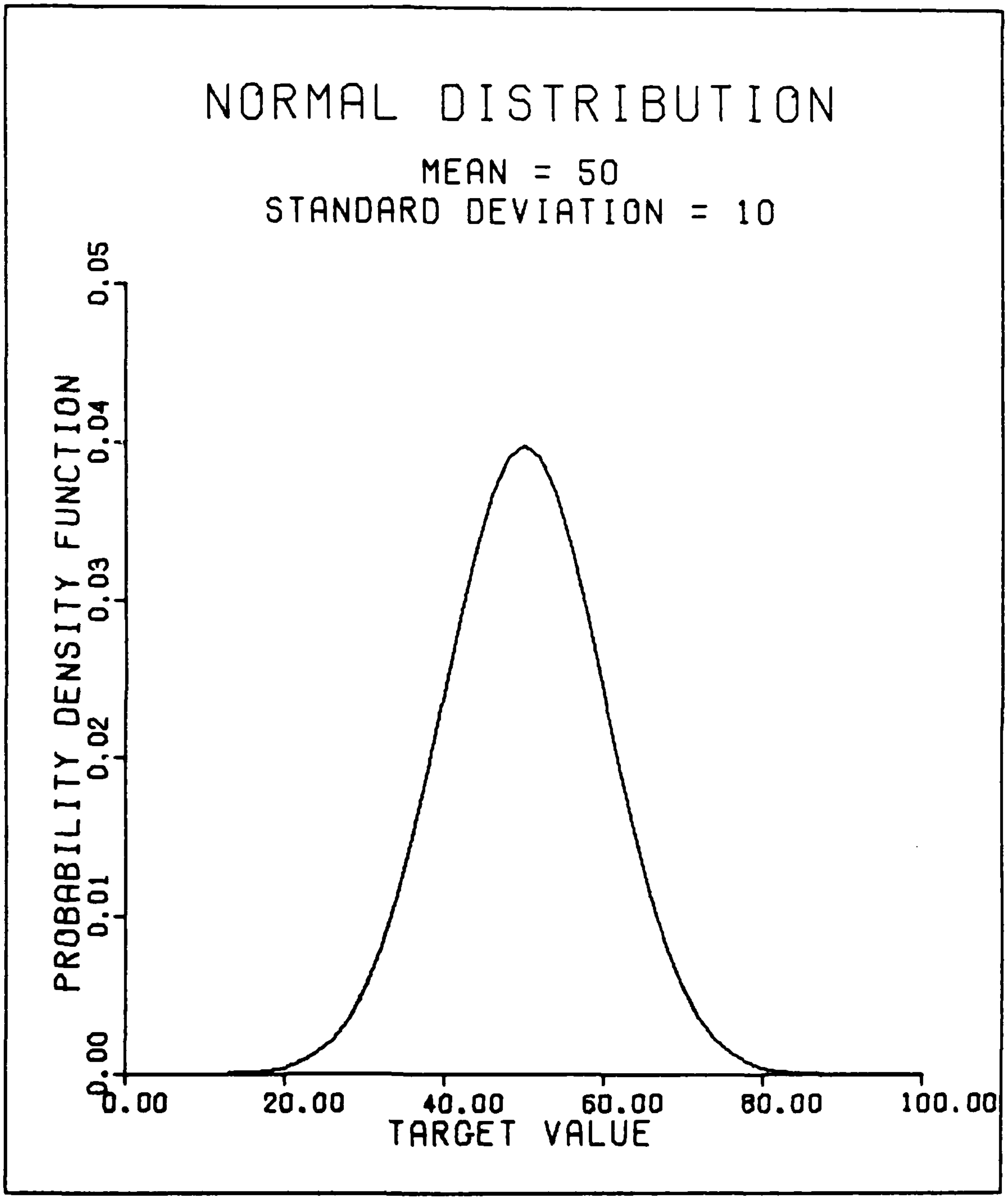

Figure 5. Truncated Low-Variance Probability Density Function

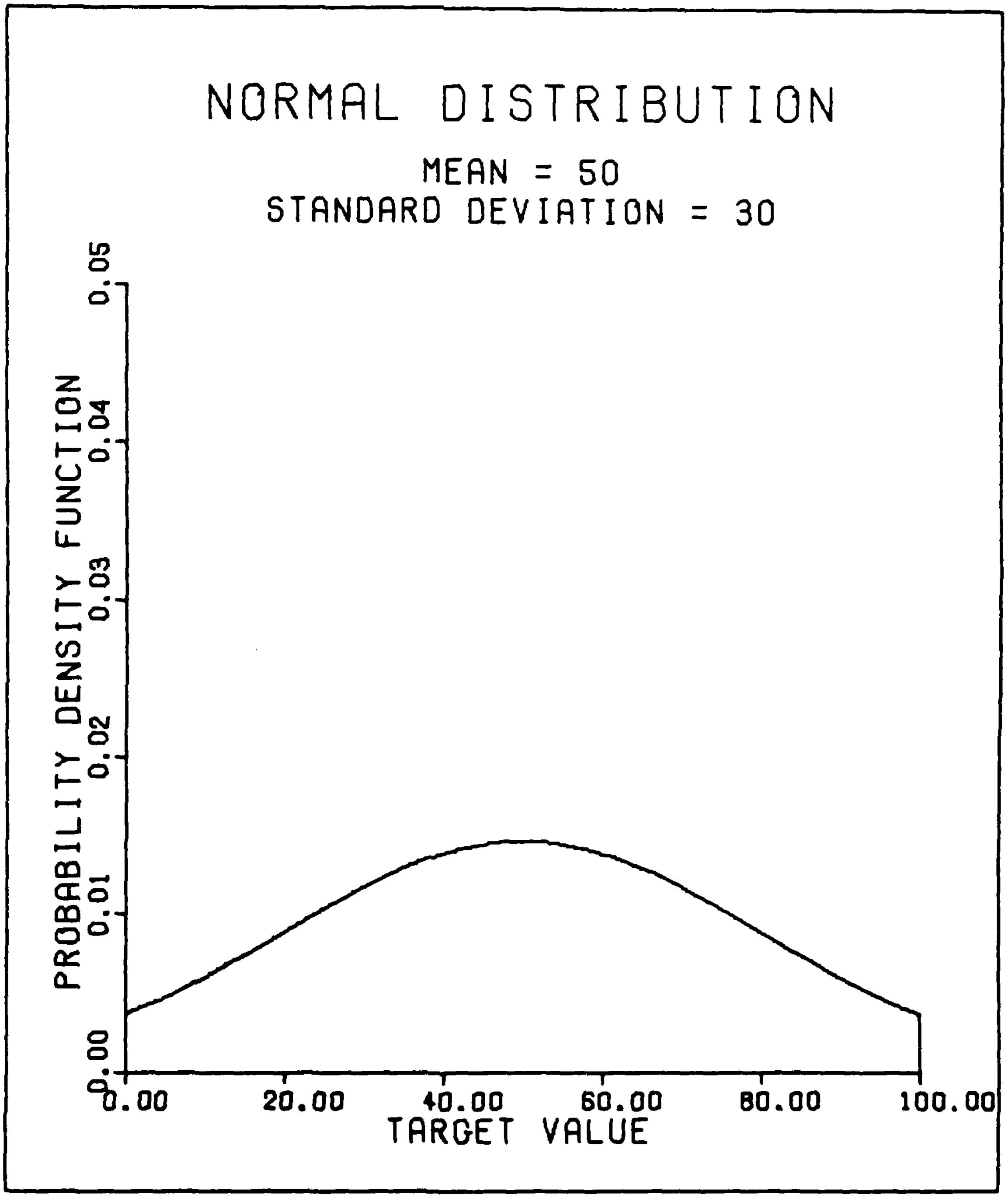

Figure 6. Truncated High-Variance Probability Density Function

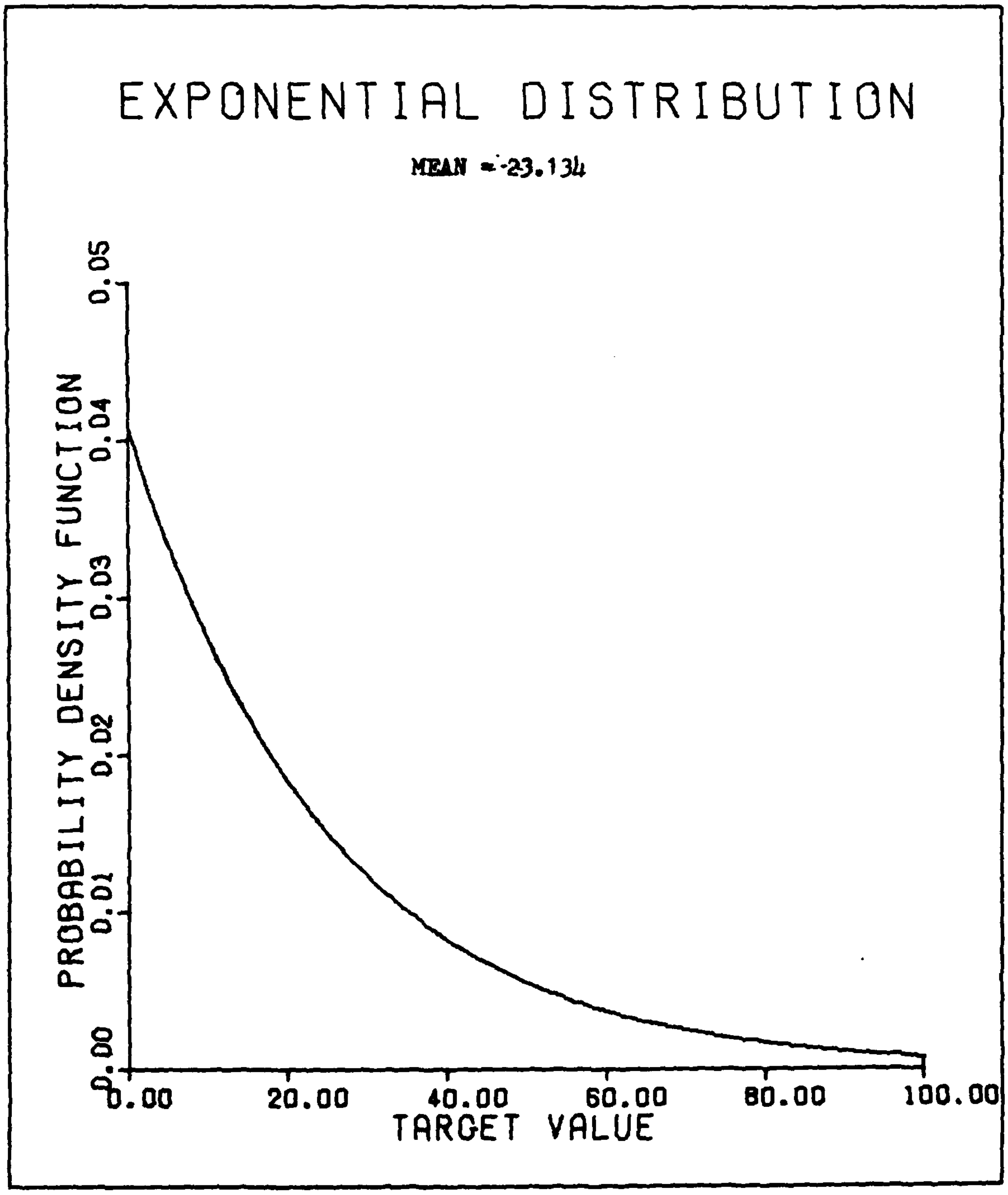

Figure 7. Truncated Exponential Probability Density Function

proportion of low-valued targets, a few medium-valued targets, and a very few targets in a high-value (above 70) category. See Figure 7 for a graph of this pdf.

Selection of Target Values. A set of two hundred random values were sampled from each distribution for a total of 1000 observations. The two hundred datapoints were divided into ten subsets of twenty independent, identically-distributed values each. Thus, a total of fifty target lists were created -- ten from each of the five distributions. (Actually all ten target lists from the integer-valued "distribution" were identical since they are selected deterministically -- the integers from one to twenty inclusive). A FORTRAN listing of the code used to generate the ten subsets of target values for the high-variance normal distribution is included as Appendix A. The target values for the other distributions were created in a similar manner by calls to the appropriate IMSL random deviate generator (GGUBFS for the uniform distribution, GGNQF for the normal distributions and GGEXN for the exponential distribution (Ref 5:G-1+)). All four sets of target values were sampled beginning with the identical random number seed -- the value shown in Appendix A. This was done to make target sets sampled from different distributions more comparable. With the identical initial random number seed for all distributions, any possible bias in the random deviates (large percentage of "big" samples, for example) would occur in all target distributions, thus reducing its effect.

Methodology

The objective of this research, as mentioned earlier, is to estimate the value of the MFEV, determine the sensitivity of the measure to changes in its input parameters and create a prediction model. Toward

that end, four factors were varied and the resulting measures of value compared. A regression equation was constructed from this data to estimate the expected value of this retargeting option, given specific values for the input parameters. All this was done twice -- once with all aircraft assigned to different targets (Phase I) and a second time when more than one aircraft is allowed to attack the same high-value target (Phase II).

Input Parameters. Two factors were treated as parameters in this research -- the number of plans in the assignment and the probability that an aircraft survives the initial enemy attack (P_s). (These variables serve as the predictor (independent) variables in the regression model). Each of these factors will be tested at four levels. Variations with two, three, four and five MFEV plans will be compared with the base case of one plan. In addition, P_s will be varied between 0.2, 0.4, 0.6, or 0.8 for each run.

The other two factors included in the model were target value distribution (five levels -- discussed earlier in this chapter) and what will be called the targeting option. This binary targeting option either 1) forces all aircraft to be allocated against different targets (and therefore forces all targets to be targeted since the number of aircraft equals the number of targets) -- Phase I; or 2) allows multiple aircraft to be assigned against the same target by the marginal value criteria presented in Chapter II -- Phase II.

Sixteen hundred datapoints resulted from 1600 runs of the model. For each of the five target value distributions, ten sets of targets were sampled. In other words, ten replications of a set of random draws from each of the five distributions (that would be fifty runs --

10 * 5) were taken. Each of these fifty divisions must be further subdivided in three ways -- by targeting options (two levels), by the number of plans (four levels), and by P_s (four levels). Thus, each of those fifty divisions were split into another 32 (2 * 4 * 4) sections leading to 1600 (10 * 5 * 2 * 4 * 4) datapoints.

Once the level of each factor was determined (specific replication from a specific target value distribution at specific value of P_s and number of plans for a given targeting option), two functions had to be accomplished so the value corresponding to this block could be measured. These two functions, assignment and simulation, are discussed in the next two sections.

Assignment. The assignment function of the model corresponds to the real-world task of building the SIOP. This portion of the model determines which targets are assigned to what aircraft in which plan -- actually a three dimensional assignment problem, as discussed in Chapters I and II.

The assignment section has one basic duty -- create a near optimal allocation of aircraft to targets over all plans. As was mentioned in Chapter I, this task is much more difficult than one might suspect at first glance. Since finding an optimal solution would normally dictate an unreasonable amount of computer resource, a heuristic solution was selected to find a near-optimal solution -- an assignment which would perform almost as well as the optimal solution, but which is more easily obtainable.

In his paper on the subject, Dimon states that in order for an assignment of targets to be optimal, the column sum of their values should be as nearly equal as possible (Ref 3:C23). The rationale behind

the concept is as follows. In order for the column sums throughout the assignment to be nearly equal, they must be worth an average of k/m where m is the number of plans. But this means the amount by which the high-value targets in the column exceed k/m is about equal to the amount by which the low-value targets fall short. Thus, for every extremely high-valued target in a column, there must be an extremely low-valued target, or two (or more) low-valued targets. But since the same targets are in every row, a high-valued target in one column means there must be lower-valued targets in the other columns. But these lower-valued targets must have higher-valued targets in the other rows of the same column in order for the columns to all sum to about k. Therefore, no matter what column might be deleted, high values will be removed from some plans and low values from others. The specific columns deleted is what determines which rows will have more high-valued targets remaining and which rows will have more low-valued targets. That row with the greatest proportion of high-valued targets would be the optimal plan, given the specific columns deleted (state of nature). For a more elaborate discussion of the rationale, see Dimon's paper (Ref 3:C7-C30).

Although Dimon's concept has great intuitive appeal and seems to work under most conditions, making the column sums as nearly equal as possible is not the best assignment in all cases. Consider, for example, four targets of value 1, 5, 12, and 25 units. Let two MFEV assignments be as shown in Figures 8 and 9. Figure 8 contains the allocation which makes the column sums as nearly equal as possible, while the optimal assignment is shown in Figure 9. The value of the two assignments is identical under all but one of the possible states of nature. When aircraft two alone is destroyed, however, the assignment obtained by

	Aircraft			
	1	2	3	4
Plan 1	25	12	5	1
Plan 2	1	12	25	5
Plan 3	5	12	1	25
Column Sum	31	36	31	31

Figure 8. Assignment Resulting From Dimon's Concept

	Aircraft			
	1	2	3	4
Plan 1	25	12	5	1
Plan 2	1	5	25	12
Plan 3	5	12	1	25
Column Sum	31	29	31	38

Figure 9. Optimal Assignment

making the column sums as nearly equal as possible (Figure 8) has a lower value than the optimal assignment (Figure 9) (31 versus 38). Thus, although intuitively reasonable, this concept can not be guaranteed to produce the optimal assignment for any particular set of target data.

Dimon, in his research, used a branch and bound technique to find the suboptimal assignment, given a specific state of nature. He then attempts to generalize these results by observing patterns in the target assignment matrix. For larger problems, even this indirect solution methodology would take an unreasonable amount of computer time to operate. Therefore, an element-interchange assignment-improvement heuristic procedure was created.

Beginning with any assignment, the procedure tests pairs of same

row elements to see if exchanging them would improve their respective column sums, i.e. would make the column sums more equal. If the exchange would be an improvement, the change is made. This test-interchange procedure continues until no additional improving exchanges can be made. At this stage, the assignment matrix is passed to the simulation section. As shown in Chapters I and IV, this assignment need not be optimal, but only near-optimal.

Simulation. Using the user supplied P_s, the simulation section generated states of nature (specific aircraft surviving an enemy attack) by means of a Monte-Carlo simulation. Once it is known which specific aircraft have survived the attack, the value of the base case and of each plan of the MFEV can be calculated. The best MFEV plan was identified and its value included in the measure of merit statistic discussed earlier in this chapter (along with the value of the base case). One thousand simulated attacks were completed on each assignment at given parameter levels. The same set of random numbers were used in the Monte-Carlo simulation for each level of all four factors (number of plans, probability of survival, targeting option and target value distribution) in an attempt to reduce the variability in the data.

Prediction. In order to create an MFEV value predictor, the 1600 datapoints created in the simulation section were used as input into the SPSS regression statistical package to produce a linear regression model (Ref 7:320-368). Ten regression equations (160 datapoints input for each) were formed, one for each of the targeting option/target value distribution pairs. In order to use the model, one would select the equation corresponding to the targeting option and target value dis-

tribution that was appropriate. After substituting the correct values for the aircraft probability of survival and the number of plans, one could compute an estimate of the value of the MFEV, given those conditions.

IV System Description -- Phase I

This chapter describes the algorithms and software package created to measure the value of the MFEV when there are an equal number of aircraft and targets and all targets must be assigned to an aircraft under each plan. Initially, the chapter describes the design of the computer model. The remaining portion of the chapter takes a step-by-step walk through the software system.

System Design

The computerization of the model is partitioned into five sections -- target selection, initialization, assignment, simulation and output. The target selection section was used to create the target values which were used in this research. The initialization section contains the program SIM which initializes variables, obtains the target values and calls all subroutines. The assignment section contains subroutines ASSIGN and RULE7. This section creates an initial assignment and iteratively improves the assignment as much as possible. The simulation section generates one thousand independent states of nature (specific sets of surviving aircraft). For each state of nature, the value of the single-plan system and the multiple-plan variation are calculated using the assignments provided by the previous section. Statistics on these values are collected. Subroutines SURV, VALUE and BASE perform the duties of this section. The output section writes data to two files -- one for visual scrutiny by the analyst and one for input into the SPSS statistical package.

The following summarizes the duties of each section, in order of execution.

Target Selection

Program MAKET was used to select the target values used throughout the research. The routine creates ten sets of twenty target values each, all sampled independently from the same distribution (one of the five used by this thesis). Essentially the same program was executed five times with only minor changes needed to control the pdf from which the values were sampled.

As the samples were taken, they were sorted into decreasing order by target value. After a set of twenty samples was taken, it was written out to a disk file. Ten such independent identically-distributed samples of twenty target values each were stored on the same file. Values from different distributions were placed on different files. Each of these files, in turn, fed its values into the master program -- Program SIM.

Initialization Section

Program SIM. Program SIM serves as a master program, initializing variables and generally setting up the remaining portion of the program for execution. The number of aircraft (which equals the number of targets) and the probability that an attacking aircraft kills a given target (PK), are always set to 20 and 1.0, respectively, for this research. The number of preplanned MFEV options and the aircraft probability of survival (PS) are then determined by user input. All of the previous values remain constant for all sets of target values taken from the same distribution. Everything after this point, though, is repeated ten times, once for each of the ten sets of target values sampled from the particular distribution. First, twenty target values are read from the data file selected by the user. Since the values were sorted before they were written on the file, the targets are entered into the target matrix

(TVALUE) in non-increasing value order (the highest value target is TVALUE(1) and the lowest TVALUE(20)). Control is then passed to the assignment section to allocate the aircraft to targets.

Assignment Section

Subroutine ASSIGN. Subroutine ASSIGN begins by assigning the base case -- the assignment against which all others are measured. Since this research only investigates the case of homogeneous bombers with identical probability of survival, the aircraft are indistinguishable. Therefore, the base case can be assigned arbitrarily since all permutations produce the identical expected value of targets destroyed. Arbitrarily, therefore, target one is assigned to aircraft one, target two to aircraft two, and so on. By the same reasoning, the first MFEV plan is identical to the base case.

The MFEV assignment can be modeled as a matrix with rows representing plans; and columns, aircraft. The value of an element stands for the value of the target assigned to that aircraft in that plan. Thus, if element (2,1) equals 17.3, then a target valued at 17.3 is assigned to aircraft one in plan two. (In the computer model, the value of an element actually represents a target number -- an index into the target value array. For ease of discussion, though, the matrix will be treated as defined above).

Once the first row (plan) has been initialized, the other rows (plans) are given an initial assignment by a simple two-stage algorithm. The first stage creates an assignment for the second row. All other rows are assigned by the second stage.

Assume the general case of n aircraft and m plans, for $m < n$. In plan two, assign target n (the lowest value target) to aircraft one.

Target $n - 1$ is then assigned to aircraft $m + 1 \pmod n$ -- the previous aircraft that was assigned (1) plus the number of plans (m), modulus n. The third lowest target is assigned to aircraft $2m + 1 \pmod n$, the previously assigned aircraft, $m + 1$, plus the number of plans (m). This modulus arithmetic assignment system continues until all targets are assigned, which is equivalent to ensuring that all aircraft are allocated. If $n = km$, where k is some integer, the modulus arithmetic has to be modified slightly so that the remainder is incremented by one when n is exceeded -- for example, $n + 1 \pmod n$ becomes 2 rather than the expected 1.

Stage two of this initial assignment procedure allocates targets to aircraft in all other plans, if there are any. Row three is created by simply shifting the row two assignment to the right by one column with wrap-around (wrapping the last column around to the first column). Row four's assignment is shifted by one more column than row three's assignment; and row five's assignment one more again. An initial assignment of five aircraft and three plans is shown in Figure 10, assuming target values of one to five, inclusive. Note that the column sums are somewhat close, as one would hope. This method was found, in most cases, to reduce run time significantly over an assignment consisting of all plans with an identical target/aircraft allocation. Consequently, the effort to improve the initial solution was worth the time required to accomplish it. Subroutine RULE1 is then called upon to improve upon this initial feasible assignment.

Subroutine RULE1. As discussed in Chapter III, the objective of the heuristic assignment procedure is to allocate the aircraft to targets so that the column sums are as nearly equal as possible. Beginning

	Aircraft				
	1	2	3	4	5
Plan 1	5	4	3	2	1
Plan 2	1	3	5	2	4
Plan 3	4	1	3	5	2
Column Sums	10	8	11	9	7

Figure 10. Sample Initial Assignment

with the initial assignment created in ASSIGN, RULE1 transposes same-row elements to monotonically advance the solution toward this goal. The column sum of column i is the sum of the target values assigned to aircraft i over all plans, as was demonstrated in Figure 10.

The approach used is to first sort the columns by column sum and then attempt to decrease the largest column sum. Elements of the column of largest sum are compared with elements of the column of smallest sum from top to bottom to see if an exchange of elements would improve the state of the column sums. Since the aircraft are homogeneous, all assignments can be made without working with row one. Therefore, row two is the first one checked and then row three and so on until a beneficial change is found. If no improvements can be made, the elements of the largest column are compared with the elements of the column with the second-smallest column sum (also in order of increasing plan number) to see if any exchange will improve the sums. If this also fails, the largest column is compared with the third smallest, the fourth smallest, and so on until an improvement can be made. If no way of decreasing the largest column sum can be found, the second largest column sum is compared with the column of smallest sum. If this leads to no improvement in sums, the second largest is compared with the second smallest and so

on as before. This method (checking the column of largest sum first against the column of the smallest and so on) was found to be more efficient than a procedure which merely began comparing arbitrary columns. However, there is no reason to suspect that this is the most efficient approach.

Once an exchange is found which would improve the column sums, the algorithm makes the exchange, determines the new column sums and resorts them. The algorithm then shifts back to begin the entire process once again (comparison of the largest column sum with the smallest). If no improvement can be made after comparing all pairs of columns, the assignment cannot be improved. This final assignment matrix and corresponding column sums are written out for the analyst. Then RULE1 passes this solution and operational control to the simulation portion of the program.

Sample Calculations. An example might be enlightening at this stage. Assume the naive initial assignment in Figure 11 has been made. The elements themselves represent target values, the rows -- plans and the columns -- aircraft.

	Aircraft			
	1	2	3	4
Plan 1	1	3	4	6
Plan 2	1	3	4	6
Plan 3	1	3	4	6
Column Sum	3	9	12	18

Figure 11. Initial Assignment

The objective of this routine, as was previously mentioned, is to make the column sums more equal. An element-interchange only improves the state of the column sums if it passes both of the following tests:

Test 1: The element of the column with the smaller sum is smaller than the element of the column with the larger sum;

Test 2: The difference between the elements must be less than the difference between the column sums.

Test 1 ensures that the larger sum doesn't get larger and, consequently, that the smaller doesn't get smaller. Test 2, on the other hand, ensures that the interchange isn't too much of a good thing. An exchange which violates Test 2 would cause the smaller column sum to become as large or larger than the large column sum and vice versa. Obviously, this would not be an improvement in the state of the column sums.

The algorithm would first compare element (2,4) with element (2,1) (the column with the largest column sum (column 4) and the column with the smallest (column 1)). Since (2,4) is greater than (2,1) (Test 1) yet not too much greater (Test 2), the exchange would be made. Figure 12 demonstrates the status after this one interchange.

	Aircraft			
	1	2	3	4
Plan 1	1	3	4	6
Plan 2	6	3	4	1
Plan 3	1	3	4	6
Column Sum	8	9	12	13

Figure 12. Solution After First Interchange

Since an exchange was made, the entire procedure begins again looking at the largest column sum (still column 4) and the smallest column sum (still column 1). Comparing elements (2,4) and (2,1) shows that (2,1) is greater so no exchange is made (Test 1 -- it would decrease

the smaller column sum and increase the larger -- exactly the opposite of the objective). Now try row 3. Element (3,4) is greater than (3,1) so it passes Test 1. But is it too large (Test 2)? Would the exchange cause the sum of column 1 to become greater than or equal to the present sum of column 4? It turns out that (3,4) is just barely too much larger than (2,1) so that, if the exchange were made, no improvement would be seen, as shown in Figure 13.

	Aircraft			
	1	2	3	4
Plan 1	1	3	4	6
Plan 2	6	3	4	1
Plan 3	6	3	4	1
Column Sum	13	9	12	8

Figure 13. Example of Unprofitable Interchange

The algorithm now calls for a comparison of the largest column (column 4) with the second smallest (column 2). Row two violates Test 1 but row three passes both tests. Therefore, elements (3,4) and (3,2) are exchanged, with the result shown in Figure 14.

	Aircraft			
	1	2	3	4
Plan 1	1	3	4	6
Plan 2	6	3	4	1
Plan 3	1	6	4	3
Column Sum	8	12	12	10

Figure 14. Solution After Two Interchanges

The column which would be labelled as having the largest column sum

(column 2 or column 3) would depend upon the sorting algorithm used. Assume column 2 was selected as largest. Since an exchange was just accomplished, the algorithm reverts to comparing the columns with the largest and smallest sums (columns 2 and 1 respectively). Row 2 fails Test 1 and row 3 fails Test 2, so column 2 is compared with column 4 (the column of largest sum with the column with the second smallest). No exchanges would be made between these columns either, because both rows fail Test 2. Next, the algorithm calls for a comparison of columns 3 and 1, the columns with the second-largest and smallest sums, respectively. Row 2 violates Test 1, but row 3 passes both tests. The exchange of (3,3) and (3,1) results in the assignment in Figure 15.

	Aircraft			
	1	2	3	4
Plan 1	1	3	4	6
Plan 2	6	3	4	1
Plan 3	4	6	1	3
Column Sum	11	12	9	10

Figure 15. Final Solution

Although this assignment is not optimal (other assignments exist in which the column sums are more equal), the algorithm, as modeled, would not perform any more exchanges. All possible interchanges would be tested and none which would produce improvement would be found. Therefore, this assignment would be passed to the simulation section.

A better solution could be achieved if one was willing to allow either 3-element swaps or exchanges which do not directly further the objective (i.e. do not change the magnitude of the column sums, but do change which particular column has a particular sum). For example,

in Figure 15, the exchange of elements (2,2) and (2,4) would result in the solution in Figure 16. Note that the column sums of column 2 and column 4 are reversed.

	Aircraft			
	1	2	3	4
Plan 1	1	3	4	6
Plan 2	6	1	4	3
Plan 3	4	6	1	3
Column Sum	11	10	9	12

Figure 16. Final Solution After Unproductive Interchange

Although no improvement was made with regards to the objective function, the optimal solution (shown in Figure 17) is obtainable in only one more interchange.

	Aircraft			
	1	2	3	4
Plan 1	1	3	4	6
Plan 2	6	1	4	3
Plan 3	4	6	3	1
Column Sum	11	10	11	10

Figure 17. Optimal Solution

This example was selected specifically to point out that this algorithm does not always find the best solution. Under most conditions, the two-way interchange procedure does eventually produce the best solution. Therefore, exchanges were limited to the pairwise switches which improve the objective function. The other types of interchanges were not allowed since they would usually increase run time dramatically without appreciably

improving the assignment determined. Besides, even the optimal solution to this problem represents only a heuristic solution to the underlying problem.

The total number of element interchanges was limited to 200 iterations by a similar rationale. The number of single-plan assignments equals the number of ways the n targets can be ordered -- $n!$. If there are m plans, there are $(n!)^m$ ways the $m * n$ assignment matrix can be arranged. But the aircraft are assumed homogeneous, so just permuting the columns should not be considered a different assignment. Similarly, if plan i and plan j were interchanged, the assignment really hasn't been changed. Therefore, the previous result should be decreased to $\frac{(n!)^m}{n!\,m!} = \frac{(n!)^{m-1}}{m!}$ possible different assignments. Although the number of possible assignments is finite, it is extremely large, as shown above. The number of possible improving exchanges, thus, although bounded, is also very large. It was discovered during the software debug phase of the research that run times were becoming excessively large for some data sets and initial assignments. Consequently, the number of allowed exchanges was limited to 200 based on the following rationale:

1) Empirically, a bound of 200 iterations impacted only a few of the assignments. The assignment iteration procedure halted in less than seventy exchanges in most cases.

2) Since the algorithm concentrates on the maximum and minimum column sums, any assignment changes made after 200 iterations should have only minimal impact on the column sums. Experimental data supported this hypothesis. When column sums derived from the algorithm limited to 200 iterations were compared with column sums from the algorithm limited to 500, the biggest change in samples

taken was less than 1%.

Once subroutine RULE1 obtains the best solution it can, control is passed from the assignment to the simulation section.

Simulation Section

The simulation section emulates one thousand independent enemy attacks on bomber bases. Given the base escape probability (equal for all aircraft in this research) defined during the initialization phase, the specific aircraft surviving each simulated attack are determined. Given this "state of nature" (specific surviving aircraft), the value of the best plan is measured and compared with the value of the base case under the same state of nature. The ratio of the averages over 1000 trials is the one data point created in the section.

The simulation section consists of three subroutines -- SURV, VALUE and BASE; each of which is called for every trial. The following describes the operation of each module.

Subroutine SURV. The purpose of subroutine SURV is to determine how many and which particular aircraft have survived the simulated missile attack. The inputs consist of the probability of survival for each aircraft defined by the user in the initialization section, and a set of random numbers. The outputs are the number of aircraft surviving and an array containing their aircraft numbers. Since the aircraft are assumed independent, the program merely compares a different uniform (0,1) pseudo random variate obtained from an IMSL routine with each aircraft's probability of survival to determine which ones live and which are destroyed. If the random number is greater than the probability of survival, that particualr aircraft is assumed destroyed. Nothing needs

to be done since a dead aircraft contributes nothing to the value of any plan. If the random number is less than or equal to the survival probability, the number of survivors in incremented and the aircraft is added to the surviving aircraft array. This procedure is accomplished for all twenty aircraft. The surviving aircraft array serves as an input for the next subroutine -- VALUE.

Subroutine VALUE. Subroutine VALUE measures the value of each plan and determines which plan is best. The inputs for this routine are the assignment generated from the assignment section and the surviving aircraft array created in subroutine SURV. The program sums the value of the targets assigned to surviving aircraft to get a measure of the value of each option (plan). A pointer is set to the plan of largest value -- the plan which should be selected by the decision maker based on the objective of maximizing expected target destruction.

Subroutine BASE. Subroutine BASE computes the value of the base case assignment and a measure of the difference between the base case and the best multiple plan value. Since plan 1 is identical to the base case, the value of the base case is exactly the value of plan 1. A non-negative measure of the improvement over the base case (delta value) is created by subtracting the value of the base case from the value of the best option. Since one of the plans is identical to the base case, a zero delta value is the worst that can be obtained. As was discussed in Chapters II and III, the value of the base case and the delta value are summed over all one thousand trials. After all the trials are completed, control is passed to the output section.

Output Section

The output section uses as inputs the sums of the base case values

and the values created by the simulation section. From these numbers, a single statistic -- percent delta value (PCDVAL) is calculated by Eq (14) derived in Chapter II. This value represents the expected increase in the value of targets destroyed under the MFEV targeting system as compared with a single-plan assignment.

PCDVAL, labelled as the "percent improvement over the base case" is added to the previous output. In addition, PCDVAL, together with two user determined parameters (the probability of aircraft survival and the number of plans), are written to another file for eventual input to the SPSS statistical package.

When all these procedures are completed for the ten sets of target values for a particular distribution, the program is complete.

V System Description -- Phase II

This chapter describes the changes to the computer software system to allow for multiple aircraft assigned against *some* targets. The chapter walks through the adapted program, giving examples to help explain the workings of the code. Emphasis is placed on differences between this version and the single-aircraft/single-target version described in the previous chapter.

The measure of effectiveness utilized in the multiple aircraft versus one target version continued to be percent improvement over the same "base case" (with all aircraft assigned against different targets), as discussed in Chapter III. The initialization and output sections were not changed at all from the previous version. In fact, only two subroutines had to be modified to allow multiple assignments against a target -- ASSIGN and VALUE. The following describes these changes and their effect on the subroutines.

Subroutine ASSIGN

As in the Phase I system (described in Chapter IV), all the initializations have been completed and the values of parameters set before entering subroutine ASSIGN. For the Phase II scenario, the duties of this module have been expanded to include the selection of targets to be attacked. It still accomplishes the original purpose of creating an initial feasible assignment. Recall that in the initial system, all targets and all aircraft were utilized; i.e. every target was attacked, so the target selection process was not needed.

The base case assignment is initialized by allocating the highest value target to aircraft number one, the second-most-valuable target to

aircraft number two, and so on. This procedure is identical to that done in the original system. Since the targets were sorted before entering this subroutine, the highest value target resides in TVALUE(1), the second-highest in TVALUE(2) and so on down the line to the least-important target (out of the twenty) in TVALUE(20), just as in the original system.

Target Selection. As stated in Chapter II, targets are assigned to aircraft by reference to the target's marginal value. In other words, each aircraft in turn is assigned against the target where it would do the greatest amount of good. Thus, if it is more valuable to send a second (or third, or ...) aircraft against a "high" value target already assigned than to hit an unassigned target, this is done. As described in Chapter II, the marginal value of target i (MV_i) is given by the following expression:

$$MV_i = P_s P_{k_i} (1 - P_s P_{k_i})^j \, V_i \tag{9}$$

where P_s = Aircraft probability of survival

P_{k_i} = Probability attacking aircraft kills target i

j = number of aircraft already assigned against target i

Since this research assumes $P_{k_i} = 1.0$ for all targets, the expression for marginal value simplifies to the following expression:

$$MV_i = P_s (1 - P_s)^j \, V_i \tag{25}$$

When the routine is entered, no aircraft have been assigned ($j=0$), so all targets have a marginal value of $MV_i = P_s V_i$. Since P_s is identical for all aircraft, the largest value target also has the largest marginal value. Therefore, the first aircraft can automatically be assigned against the highest value target. Then the marginal value

of target one is updated ($MV_1 = P_s(1 - P_s)^1 V_1$) and placed into its location in the ordered marginal value vector. If V_1 is much larger than V_2, it may be that target one still has the largest marginal value. If so, aircraft two will also be assigned to target one. If, on the other hand, $MV_2 = P_s V_2 > P_s(1 - P_s) V_1 = MV_1$, then aircraft two will be allocated against target two. Whichever path is taken, the marginal value of the target against which aircraft two is assigned is then updated and resorted into the marginal value array. Then aircraft three is assigned against the target with the largest marginal value. This procedure continues until all twenty aircraft are assigned to targets.

Sample Calculation. As an example, assume four aircraft and four targets of value equal to one, three, four, and six. Thus, the highest value target is worth six units and the least-important target is only worth one-sixth as much or only one unit.

If we assume that $P_s = .6$ *and* $P_k = 1.0$ *for this example, the* initial residual and marginal values are given in Figure 18. Residual value is the expected value of the target if no additional aircraft attack it, and is shown only for illustrative purposes.

	Target 1	2	3	4
Residual Value	6.0	4.0	3.0	1.0
Marginal Value	3.6	2.4	1.8	0.6

Figure 18. Initial Data for Sample Calculation

Note that since no aircraft have yet been assigned against any of the targets, the residual value of each target equals its full value. The marginal value is obtained by multiplying the probability of survival

times the target value and equals the change in residual value if one more aircraft is assigned against that target. The first aircraft is assigned to target one. Figure 19 gives the residual and marginal values after this one assignment. The assignment status is shown in Figure 20.

	Target			
	1	2	3	4
Residual Value	2.4	4.0	3.0	1.0
Marginal Value	1.44	2.4	1.8	0.6

Figure 19. Post First-Assignment Values

	Aircraft			
	1	2	3	4
Target	1	x	x	x

Figure 20. Post First-Assignment Status

Note that both the residual and marginal values of target one are updated by multiplication by $(1 - P_s)$. Target two now has the largest marginal value (2.4), so aircraft two is assigned against it. These results are shown in Figures 21 and 22.

	Target			
	1	2	3	4
Residual Value	2.4	1.6	3.0	1.0
Marginal Value	1.44	.96	1.8	0.6

Figure 21. Post Second-Assignment Values

	Aircraft			
	1	2	3	4
Target	1	2	x	x

Figure 22. Post Second-Assignment Status

Next, aircraft three must be assigned. Since target three has the largest marginal value, aircraft three should be allocated to it. The outcome of this assignment is shown in Figures 23 and 24.

	Target			
	1	2	3	4
Residual Value	2.4	1.6	1.2	1.0
Marginal Value	1.44	.96	.72	0.6

Figure 23. Post Third-Assignment Values

	Aircraft			
	1	2	3	4
Target	1	2	3	x

Figure 24. Post Third-Assignment Status

After these three assignments, target one has the highest marginal value so aircraft four is allocated to target one. The resulting final assignment for this simplified problem is given in Figure 25.

	Aircraft			
	1	2	3	4
Target	1	2	3	1

Figure 25. Final Assignment Status

		Aircraft		
	1	2	3	4
Target	1	2	x	x

Figure 22. Post Second-Assignment Status

Next, aircraft three must be assigned. Since target three has the largest marginal value, aircraft three should be allocated to it. The outcome of this assignment is shown in Figures 23 and 24.

		Target		
	1	2	3	4
Residual Value	2.4	1.6	1.2	1.0
Marginal Value	1.44	.96	.72	0.6

Figure 23. Post Third-Assignment Values

		Aircraft		
	1	2	3	4
Target	1	2	3	x

Figure 24. Post Third-Assignment Status

After these three assignments, target one has the highest marginal value so aircraft four is allocated to target one. The resulting final assignment for this simplified problem is given in Figure 25.

		Aircraft		
	1	2	3	4
Target	1	2	3	1

Figure 25. Final Assignment Status

Although aircraft one and four are assigned against target one in the final solution, this need not be the case. Any assignment which allocates two aircraft to target one and one aircraft each to targets two and three would yield an identical expected value and thus would be equally good. As is apparent by referring to the marginal values in Figure 23, if another aircraft was available, it would be allocated against target two.

Assignment Selection. Once the decision is made on the number of times each target will be assigned in a given plan by the method demonstrated above, an initial feasible solution is constructed. The result of the algorithm just analyzed becomes plan one. The elements of this target list are permuted and placed in the other plans by the same algorithm as in Phase I to serve as a starting point for the iterative allocation scheme. Then, this initial assignment is passed to subroutine RULE1 which determines a final solution by the same method described in Chapter IV.

Once the assignment is made, the simulation subsection takes over. For each of the 1000 trials, subroutine SURV is called to determine which aircraft survive the enemy attack, exactly as in the previous Phase I system. Then an altered subroutine VALUE is called in order to measure the target destruction capability of the MFEV, given the assignment determined in RULE1 and specific surviving aircraft selected in SURV.

In order to make the Phase I and Phase II results even more comparable, the same random number seed was used for each phase -- that is, the same aircraft survived each trial in each phase.

Subroutine VALUE

The duties of subroutine VALUE are unchanged from the previous

system. Now that the same target may be attacked numerous times in the same plan, full credit for destroying a target cannot be given to each aircraft attacking it. Rather, the marginal value -- the value expected to result from sending an additional aircraft against a target -- would be the appropriate measure. The following expression for the value of a plan was derived in Chapter II for use in evaluating the contribution of surviving aircraft at the target's marginal value:

$$E(\text{Value of targets destroyed}) = \sum_{i=1}^{n} X_i \sum_{j=1}^{m_i} (P_{k_i}(1 - P_{k_i})^{j-1} V_i) \tag{11}$$

where n = number of targets

m_i = the number of surviving aircraft assigned against target i in this plan

P_{k_i} = probability that an attacking aircraft destroys target i

V_i = value of target i

X_i = 1, if $m_i \geq 1$
= 0, otherwise

However, in this research, $P_{k_i} = 1.0$ for all i. Therefore, the marginal value of the first surviving aircraft assigned against any target i is V_i and any additional surviving aircraft assigned against the same target do not increase the value of the plan (marginal value = 0.0). So the value of a plan equals the sum of the values of the targets assigned at least once to a surviving aircraft in that plan.

For example, assume the multiple-plan assignment given in Figure 26 where a matrix element represents the target number assigned. Let four (numbers two, four, five and six) out of the original eight aircraft survive the enemy attack. Under these conditions and the assumption that $V_1 \geq V_2 \geq V_3 \geq V_4 \geq V_5 \geq 0$, it is easily verified that Plan 2 is the

	Aircraft 1	2	3	4	5	6	7	8
Plan 1	1	2	3	1	4	2	5	6
Plan 2	5	3	1	2	1	6	4	2
Plan 3	2	5	4	6	3	1	1	2

Figure 26. Aircraft Assignment

	Surviving Aircraft 2	4	5	6
Plan 1	2	1	4	2
Plan 2	3	2	1	6
Plan 3	5	6	3	1

$$\text{Value (Plan 1)} = V_2 + V_1 + V_4 + 0 = V_1 + V_2 + V_4$$

$$\text{Value (Plan 2)} = V_3 + V_2 + V_1 + V_6 = V_1 + V_2 + V_3 + V_6$$

$$\text{Value (Plan 3)} = V_5 + V_6 + V_3 + V_1 = V_1 + V_3 + V_5 + V_6$$

Figure 27. Phase II Subroutine VALUE Example

preferred plan, as shown in Figure 27.

Once each plan is evaluated and the best one identified, subroutine VALUE returns control back to program SIM. Subroutine BASE is then called and the same data collected as under the Phase I system. This entire process is repeated over the 1000 trials and the same statistics calculated and output as under the original design.

VI Analysis

The 1600 datapoints from the simulation section, together with the target values which were used to create them, served as input for the analysis. Four different approaches were taken to view the results from slightly different angles. These approaches were: 1) to average the data over the ten random draws from each distribution which determined the sets of target values; 2) to determine a multiple linear regression model relating improvement over the value of the basic solution to the aircraft survival probability (P_s) and number of preplanned MFEV plans for a given target value distribution and targeting option; 3) to compare these findings against results derived from the best possible solution -- perfect advance information about the state of nature (specific aircraft surviving); and 4) to compare the best possible solution to the maximum value obtainable if all aircraft survived the attack.

Data Averages

As was explained in Chapter III, ten independent sets of target values were sampled from each of the five distributions. The different target sets were then used as inputs to the model at a specific set of parameter levels to obtain ten estimates of the same value. These ten data points were averaged to obtain the 160 elements in Tables I through X. These tables can be thought of as a four dimensional matrix since four parameters vary among the tables: 1) the aircraft survival probability (four levels); 2) the number of MFEV plans (four levels); 3) target value distribution (five levels); and 4) targeting option (two levels). The values shown represent the average

Table I

Percent Improvement -- Integer Distribution, Phase I

		Survival Probability			
		.2	.4	.6	.8
Number of MFEV Plans	2	19.53	11.96	7.96	4.89
	3	25.22	15.42	10.41	6.30
	4	28.77	17.82	11.86	7.21
	5	31.05	19.24	12.85	7.80

Table II

Percent Improvement -- Uniform Distribution, Phase I

		Survival Probability			
		.2	.4	.6	.8
Number of MFEV Plans	2	20.67	12.64	8.42	5.15
	3	27.08	16.52	11.09	6.74
	4	30.49	19.10	12.71	7.69
	5	33.49	20.76	13.87	8.30

improvement over the base case for the ten sets of target values.

Tables I through V represent results when all targets were assigned in each plan. The individual tables differ only in the distribution from which the target values were sampled. The number of preplanned MFEV options varies from two to five and the aircraft survival probability is sampled at .2, .4, .6 and .8. Tables VI through X represent the identical measures for Phase II of the research, where all targets need not be assigned in each plan.

Survival Probability Effects. Within each of the ten tables, two consistent trends are obvious. First (moving across any row), the value of the MFEV decreases as survival probability increases. This is consistent with intuition since as P_s increases, more and more aircraft survive, so more and more of the higher value targets would be targeted

Table III

Percent Improvement -- Low-Variance Normal Distribution, Phase I

Number of MFEV Plans	Survival Probability .2	.4	.6	.8
2	6.82	4.22	2.80	1.70
3	8.93	5.49	3.67	2.22
4	10.17	6.31	4.22	2.53
5	11.11	6.88	4.58	2.78

Table IV

Percent Improvement -- High-Variance Normal Distribution, Phase I

Number of MFEV Plans	Survival Probability .2	.4	.6	.8
2	16.59	10.23	6.80	4.13
3	21.66	13.50	9.05	5.44
4	24.84	15.44	10.24	6.19
5	27.02	16.77	11.24	6.75

Table V

Percent Improvement -- Exponential Distribution, Phase I

Number of MFEV Plans	Survival Probability .2	.4	.6	.8
2	29.58	18.34	12.13	7.37
3	42.67	25.66	16.80	9.64
4	50.59	29.82	19.20	10.75
5	55.26	32.71	20.90	11.58

in the base case. Therefore, there is less and less value in having the flexibility to select from a variety of plans.

Number of Plans Effect. Secondly, as the number of plans is increased, the value of the MFEV system also increases. This is consistent with preconceived notions, too, since increasing the number of preplanned options only increases the variety of plans available to the

Table VI

Percent Improvement -- Integer Distribution, Phase II

		Survival Probability			
		.2	.4	.6	.8
Number of MFEV Plans	2	46.34	26.51	15.46	5.03
	3	53.59	31.88	17.28	7.47
	4	56.81	34.04	19.08	8.27
	5	58.38	35.45	19.84	8.57

Table VII

Percent Improvement -- Uniform Distribution, Phase II

		Survival Probability			
		.2	.4	.6	.8
Number of MFEV Plans	2	50.10	29.40	16.71	6.70
	3	56.16	34.50	19.67	8.27
	4	59.98	37.13	21.65	9.17
	5	61.85	38.77	22.56	9.88

decision maker, thereby increasing the expected value of the plan he does select.

<u>Target Value Distribution Effect</u>. A few other trends are less obvious from viewing these tables, but exist nonetheless. Comparing corresponding elements of the first five tables shows a remarkable variance between values measured from different target value distributions. Three of the tables show a similarly-sized value (integer, uniform, and high-variance normal distributions), but the low-variance normal and exponential distributions are much different. The low-variance normal has smaller values than the other distributions and the exponential has higher values. The magnitude of the difference varies somewhat with the number of plans and survival probability, but the trend remains intact. A similar effect can be found between the same elements

Table VIII

Percent Improvement -- Low-Variance Normal Distribution, Phase II

Number of MFEV Plans	Survival Probability .2	.4	.6	.8
2	12.80	5.26	2.67	1.70
3	15.74	7.05	3.66	2.23
4	17.28	8.05	4.20	2.52
5	18.28	8.82	4.65	2.73

Table IX

Percent Improvement -- High-Variance Normal Distribution, Phase II

Number of MFEV Plans	Survival Probability .2	.4	.6	.8
2	38.16	21.51	11.11	3.82
3	43.91	25.49	13.52	5.51
4	46.96	27.73	14.85	6.14
5	48.81	29.19	15.84	7.00

Table X

Percent Improvement -- Exponential Distribution, Phase II

Number of MFEV Plans	Survival Probability .2	.4	.6	.8
2	94.00	50.72	26.05	10.71
3	105.28	56.99	30.05	12.36
4	110.22	60.01	31.66	13.08
5	114.73	62.12	32.97	13.77

of the second five tables -- the Phase II results.

The discovery that the value of the MFEV system is highly dependent upon the target value distribution can be readily explained. In the range (1, 20), the integer-valued distribution can be thought of as almost uniform. The mean is 10.5 ((1 + 2 + ... + 20)/20) which is almost identical to a uniform (0, 20) distribution (mean = 20/2 = 10).

Also, the values are equally spread out in the region. Since the integer-valued target distribution is very much similar to a uniform distribution, it makes sense that the values measured for the two distributions would be very much alike. A similar argument can be used for the high-variance normal distribution used in this study. As was shown in the graph of the normal (50, 30) pdf in Figure 6 in Chapter III, one would expect twenty target values sampled from this distribution to be fairly uniform also (with a slight central tendency). Thus, it is not at all surprising to discover that the high-variance normal, uniform, and integer distributions have similar value measurements. Even the finding of slightly lower values for the high-variance normal distribution is consistent with this argument, due to the central tendency inherent in any normal distribution.

The scores achieved by the low-variance normal and exponential distributions are also readily explained by comparison with the uniform distribution. Since, in a normal (50, 10) distribution, most of the target values are expected to be grouped in a small region near the mean, little benefit can be achieved by exchanging a higher-value target for a lower-value one. That is, since one would expect little difference between the values of two arbitrarily chosen targets, the benefit earned by creating the flexibility to select between them is small. Thus, the value of the MFEV system under the assumption of low-variance normal target values is much smaller than under any other assumption of target values (approximately one-third of the value measured under the assumption of uniform target values).

The assumption of exponential target values, on the other hand, yielded much higher scores than any other target value distribution (about twice the value of uniform distribution). Under this distribution,

most of the targets are expected to be of very low value ($\leq$30 units), a few in the medium range (value 30 to 70 units) and perhaps a very few targets of high value. Therefore, for a given state of nature, most of the aircraft which survive an enemy attack would be expected to be aimed against medium or low-valued targets in the base case. Thus, the value of the MFEV multiple-plan system under the assumption of exponential target values should be higher than under any other distribution studied in this research.

Targeting Option Effect. A targeting option effect becomes apparent if one compares results between the two phases of this research. Holding everything else constant, the Phase II results can be seen to be larger than the Phase I findings (i.e. comparison of the same matrix element of the same target distribution table between the two phases). This effect is highly dependent on P_s, however. The difference is greatest for small P_s (the value for Phase II was found to be on the order of twice the Phase I value), but the values rapidly converge as P_s increases, until they are very nearly equal at $P_s = .8$. A strong interaction between P_s and targeting option makes good intuitive sense. For large P_s, as discussed in Chapter II, the marginal value of any target i is given by the following equation:

$$MV_i = P_s P_{k_i} (1 - P_s P_{k_i})^j V_i \tag{9}$$

Thus, for $P_s = .8$, the marginal value of a given target rapidly becomes small as additional aircraft are allocated to it. (Since $P_k = 1.0$ for all i, the marginal value is decreased by eighty percent each time another aircraft is assigned to it). Therefore, at a P_s of .8, few targets have more than one aircraft allocated to them. But Phase I

differs from Phase II only in that no target has more than one aircraft assigned against it. Thus, at high P_s, the two phases will have similar assignments and, therefore, should have similar values.

The fact that the Phase II values are consistently higher than the Phase I measures is also comforting. Since Phase II is merely a more flexible version of Phase I, multiple aircraft would not be assigned against a target unless it would increase the expected value of targets destroyed. Thus, the expected value of a Phase II solution should always be greater than the expected value of a Phase I assignment. The validity of the model would have been suspect had not this result been confirmed.

Regression Model

Tables of data, although informative, often fail to convey the overall picture of any complex situation. Therefore, the same datapoints from which the tables were compiled were used as inputs to the SPSS Regression routines. The datapoints were used to fit a model of the following form for each distribution/phase combination:

$$\text{Percent Improvement} = b_0 + b_1 P_s + b_2 P_s^2 + b_3 N$$

The least squares estimates of the above coefficients (b_0, b_1, b_2, and b_3) are listed in Tables XI and XII. Table XI pertains to the Phase I data and Table XII, Phase II. For both tables, the predictor (independent) variables are number of preplanned MFEV options (N), the aircraft probability of survival (P_s) and the probability of survival squared (P_s^2). The criterion (dependent) variable is the expected percent improvement over the base case (single-plan system with all targets assigned in each plan).

Table XI

Phase I Regression Coeffecients

	N	P_s	P_s^2	Const	Coef of Det (R^2)
Integer	2.203	- 68.377	36.319	30.477	.964
Uniform	2.439	- 73.385	38.942	32.422	.845
Low-Variance Normal	0.809	- 24.026	12.645	10.664	.846
High-Variance Normal	1.979	- 57.919	30.244	25.831	.854
Exponential	4.366	-122.172	65.446	50.735	.843

Table XII

Phase II Regression Coefficients

	N	P_s	P_s^2	Const	Coef of Det (R^2)
Integer	2.368	-146.873	70.176	71.848	.990
Uniform	2.496	-145.360	65.183	74.545	.828
Low-Variance Normal	0.989	- 67.522	45.177	24.098	.868
High-Variance Normal	2.150	-128.515	64.177	56.950	.849
Exponential	3.416	-347.126	193.124	155.214	.887

As can be seen from the tables almost 85 percent of the variability in the data can be explained by the regression equations in the four random target value distributions. The deterministically-selected fifth distribution (successive integer target values) has an R^2 above 0.96 in each case.

Figures 28 and 29 are sample plots of four regression curves together with the data from which they were derived. Figure 28 contains the Phase I curves from the exponential distribution at $N = 3$ and $N = 5$ while Figure 29 contains the same information from the Phase II

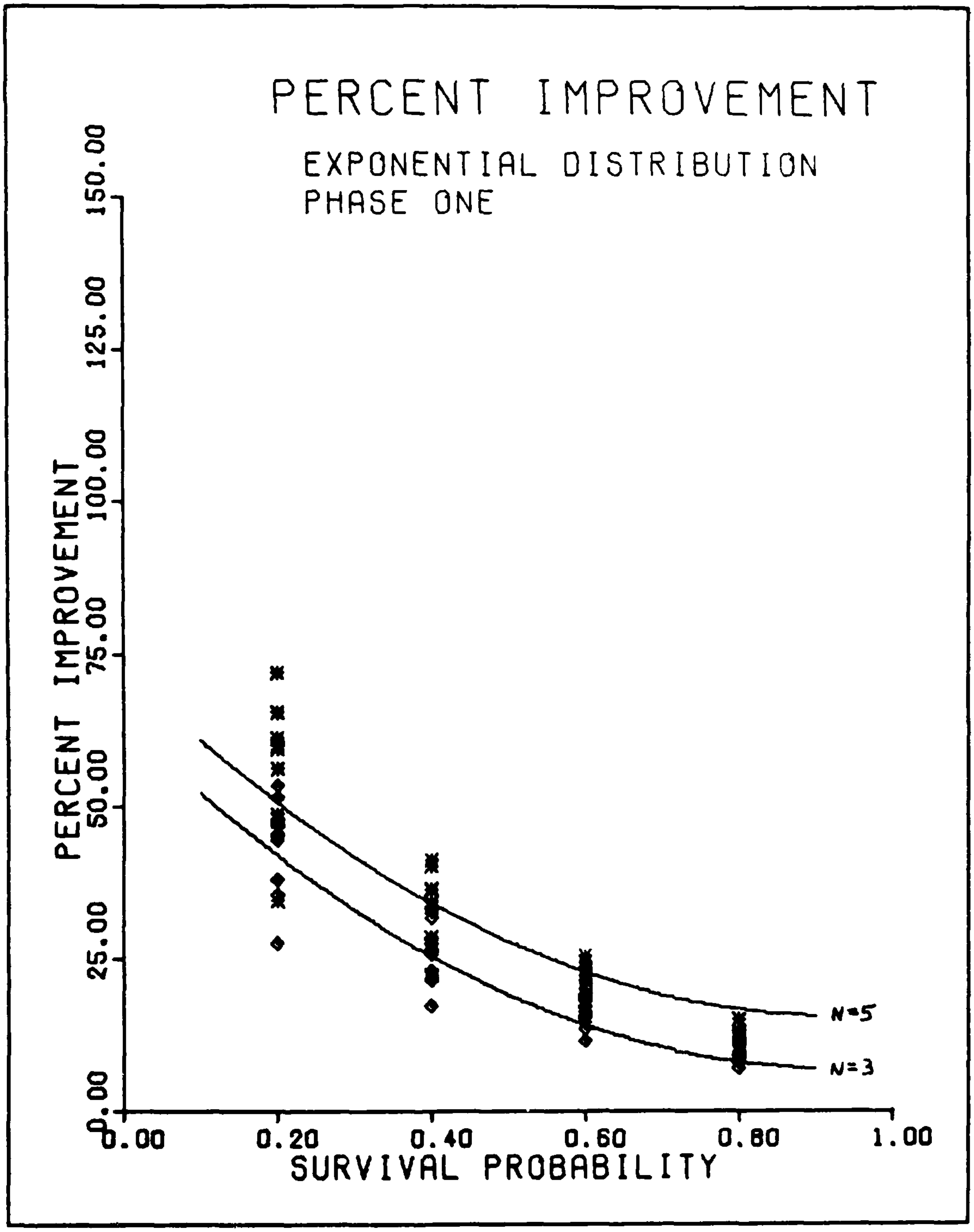

Figure 28. Sample Regression Curves, With Data -- Phase I

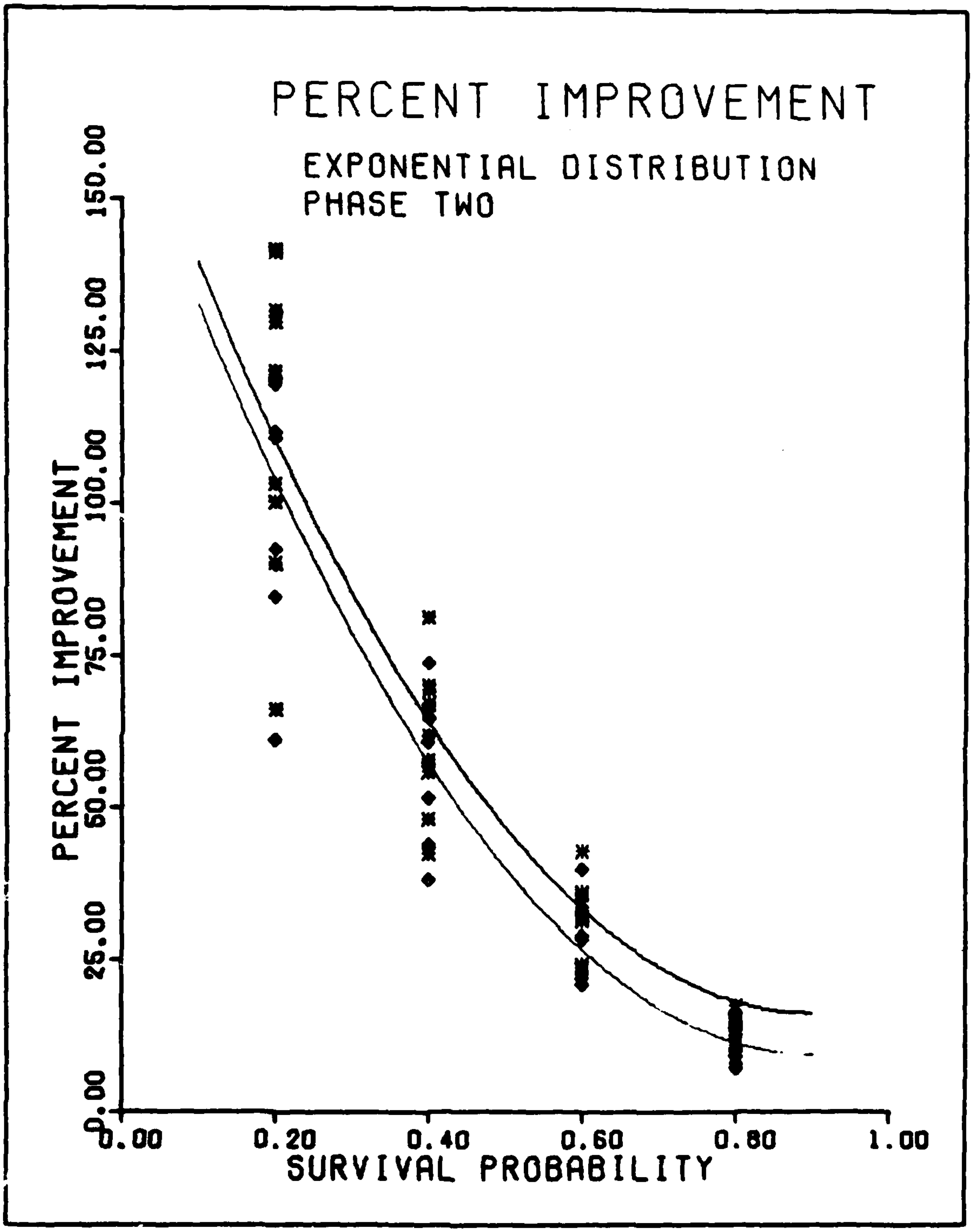

Figure 29. Sample Regression Curves, With Data -- Phase II

data.

All the regression curves are shown in Figures 30 through 34. The solid lines represent Phase I data and the dashed lines are derived from the Phase II results. Each figure represents 320 datapoints -- forty for each of the eight curves. A single curve shows the percent improvement over the base case for fixed target value distribution, targeting option and number of preplanned MFEV options as a function of aircraft probability of survival.

Effect of Survival Probability. As can be seen from the graphs, survival probability is the biggest determinant of MFEV value. For small values of P_s ($P_s = .2$), the value of the MFEV is about four times as great as for large P_s values ($P_s = .8$) under Phase I assumptions and about ten times as great when the Phase II figures are compared.

The value of the MFEV is strictly greater than zero for any $P_s < 1.0$ and any selection of factor levels. The slopes of the regression curves, however, as shown in Figures 30 - 34, approach zero for large values of P_s. Under four of the five target value distributions, though, as P_s decreases past .8, the value of the MFEV increases rapidly. Many studies have attempted to estimate P_s (or a simialr measure of survival probability) with some variety of conclusions. The results of the research documented herein would indicate that, especially for $P_s < .6$, the benefit of an MFEV system might very well outweigh the costs of implementing it. Of course, additional analysis involving operational targets and constraints would have to be conducted, and some measure of quantifying the value added by the MFEV created, before a cost-benefit analysis to compare this value with the economic/political costs involved

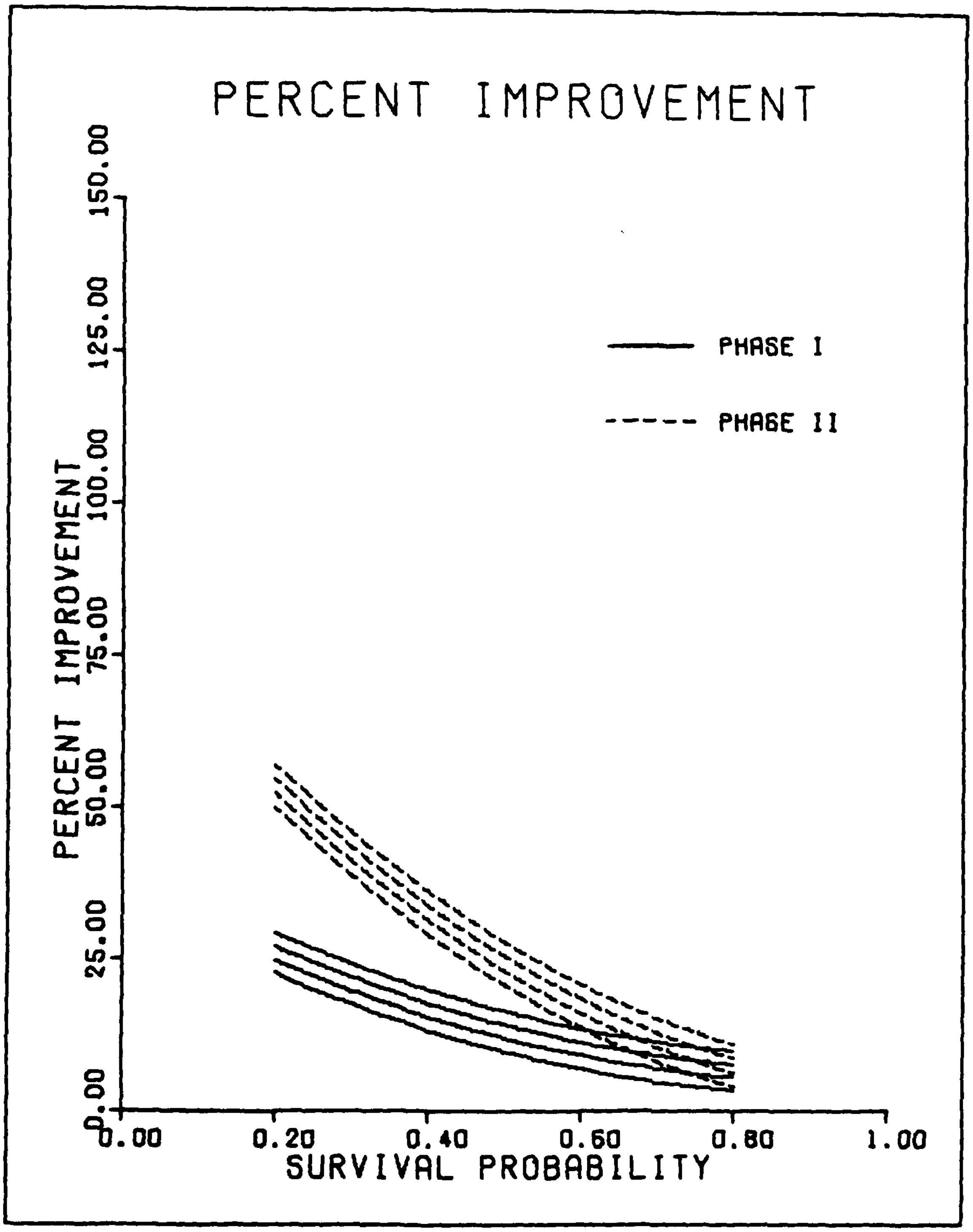

Figure 30. Integer Distribution Regression Curves

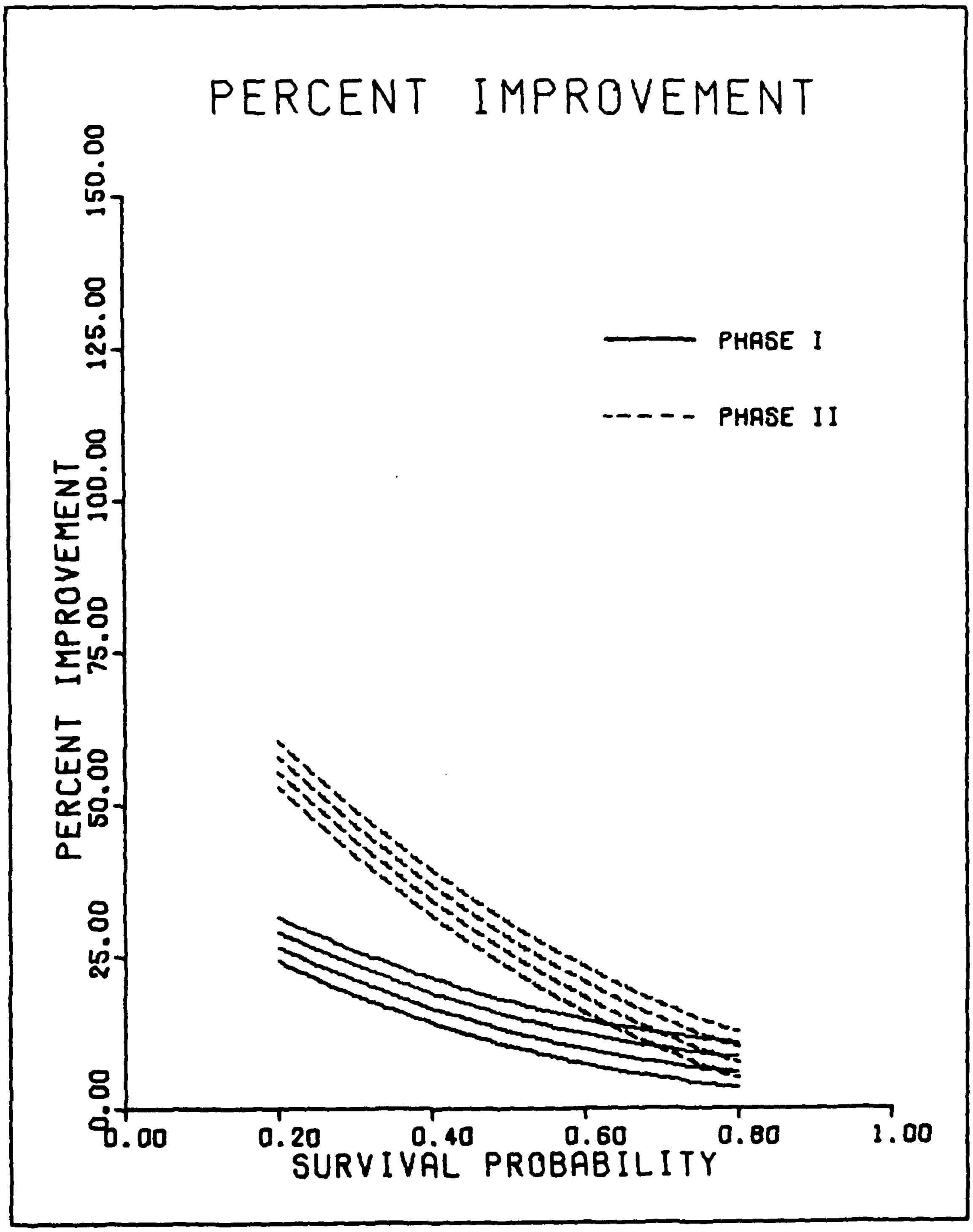

Figure 31. Uniform Distribution Regression Curves

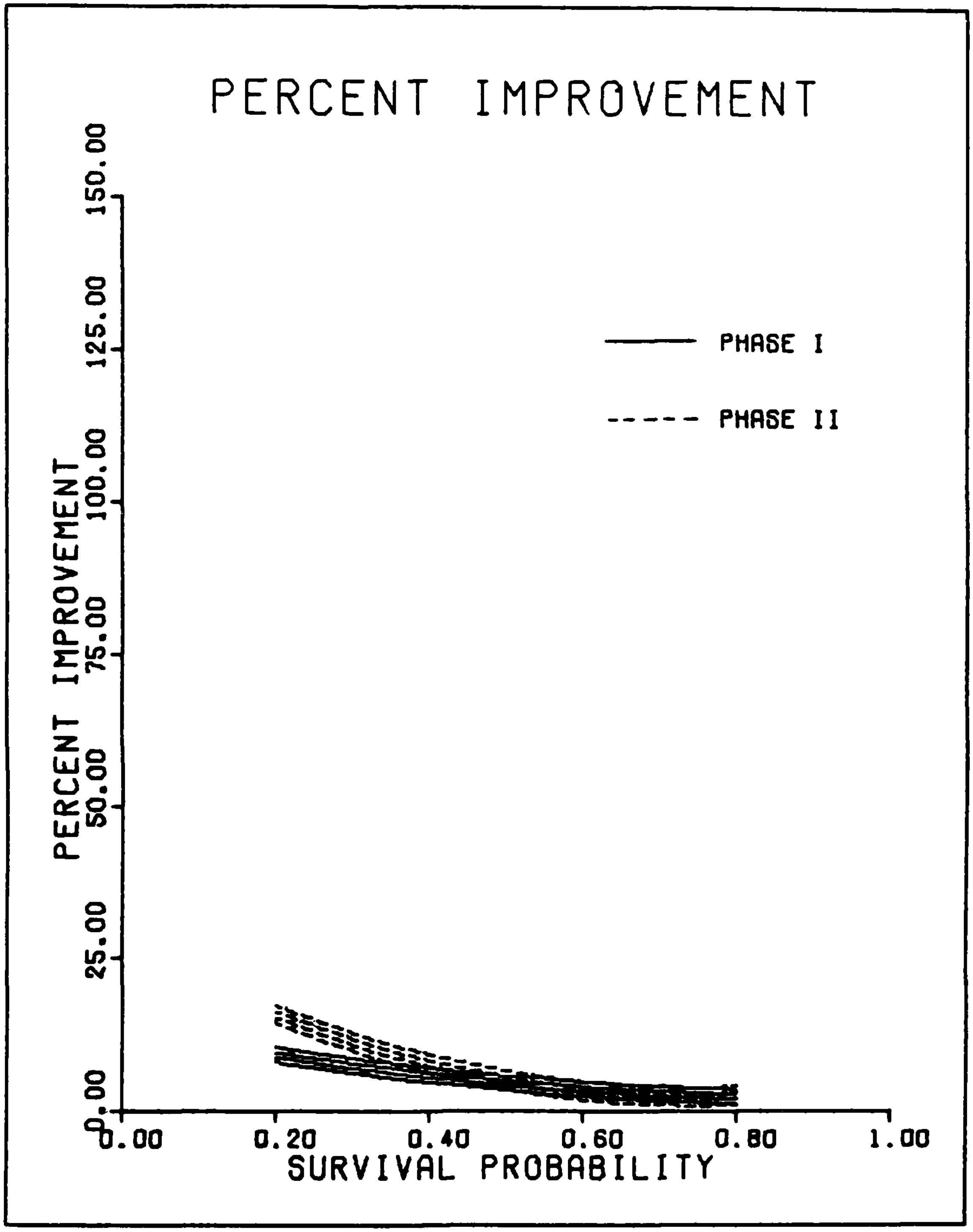

Figure 32. Low-Variance Normal Distribution Regression Curves

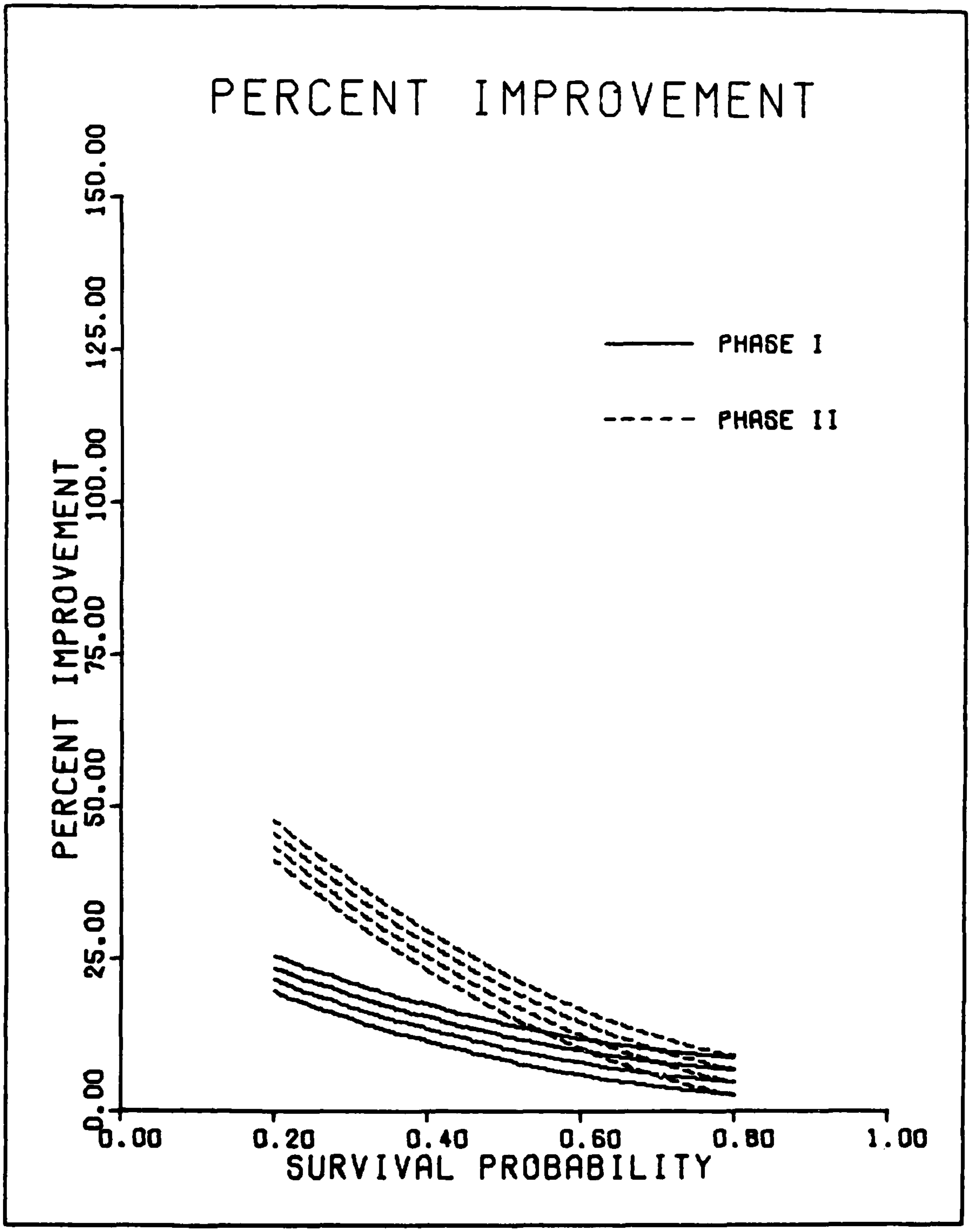

Figure 33. High-Variance Normal Distribution Regression Curves

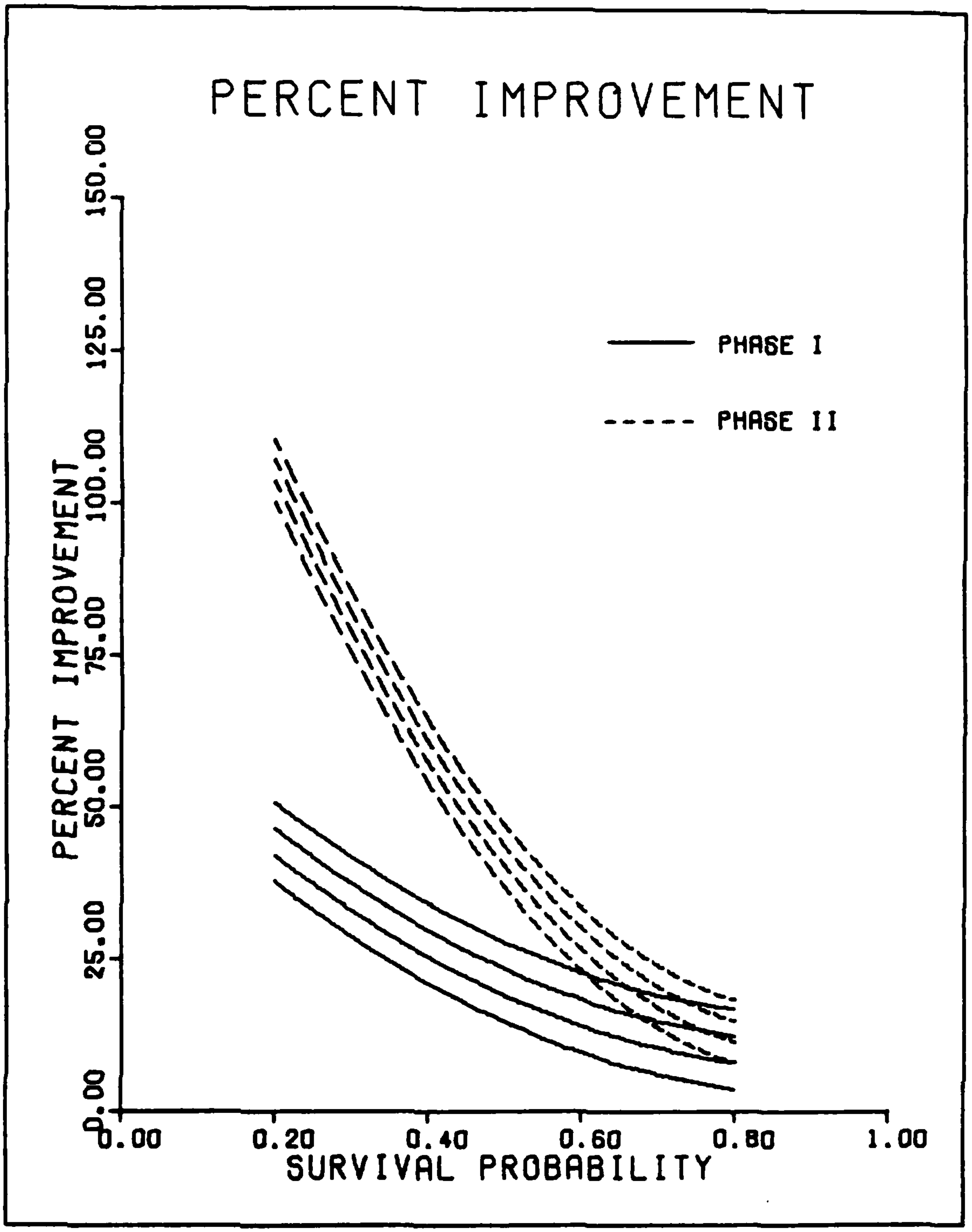

Figure 34. Exponential Distribution Regression Curves

could be accomplished.

Effect of the Number of Plans. The number of plans in the MFEV system was found to be only a minor factor in determining the value of the MFEV. An increase in the number of plans did, in every case, increase the value, but only by a few percentage points. Diminishing marginal returns seemed to be present -- as the number of plans was increased from two to three, three to four, and four to five; less and less value seemed to be added by incorporation of another plan. A quadratic term (N^2) was added to the previously mentioned regression model to test for diminishing marginal returns of the number of plans. The addition of the term, however, caused the variance of the coefficients of both N and N^2 to become very large, indicating a high degree of collinearity between the two terms. Thus, although N^2 could explain some of the variance in the data, it could not add much to the model when N was already included linearly. Therefore, N was included in the model solely as a linear term.

Effect of Target Value Distribution. Essentially the same findings can be seen in the regression curves as were discussed in the Data Averages section of this chapter. Target value distribution was found to be a fairly important factor in approximating MFEV value. The corresponding curves from all distributions had similar shapes. In fact, they seemed to be the same curve plotted five times with a different scale factor on the value.

The curves for three of the distributions (integer, uniform and high-variance normal) were very much alike, for all levels of all factors. The other two target value distributions tested in this research were graphed as well above (exponential), and well below (low-variance normal) the middle three curves.

Thus far, this research has only compared the MFEV with a targeting system similar to the SIOP. One might well ask how the MFEV compares with an airborne retargeting system of even greater flexibility. The next section does just that.

<u>Comparison with Best Possible Solution</u>

The MFEV values were compared with the analytically-determined expected values of the best possible solution (perfect prior-attack information). Another way to look at this best possible solution case is to assume all aircraft have perfect command and control. Under this assumption, any k surviving aircraft will be able to strike the k highest-value targets. Thus, for a given state of nature (number of aircraft surviving), such an assignment is the best possible solution. The value measured is the percentage of this "best" solution which can be achieved by the MFEV under the variety of conditions studied. This measure utilizes the "percent of total value" measure of effectiveness introduced by Thomas in his commentary (Ref 9:2) of Dimon's original work and is discussed in Chapter III and below.

The results achieved by comparing the MFEV value with the best possible solution are shown in Figures 35 - 39. Each figure represents one target value distribution and graphs the percentage of the best possible solution against aircraft probability of survival. The eight datapoints at each level of P_s correspond to the four levels of the number of plans factor for each phase -- four from Phase I and four from Phase II. The values which define the solid curve are the percentage of the best possible solution achieved by the base case (as derived in Chapter II -- Eq (24)).

Since all MFEV values for a given distribution and P_s level are

compared with the same best possible solution figure, the same relative positions would be expected to hold as did in the percent improvement over the base case measure. Thus, increasing the number of plans resulted in an increase in the value of the system. Again, Phase II measurements are always at least as good as Phase I values. After these similarities, the pattern changes, however. The low-variance normal value is slightly higher than any other value which is the opposite of the percent improvement over the base case. Similarly, the exponential values tend to be lower than values from the other distributions. The integer, uniform and high-variance normal measures, however, are once again very much similar to each other and are grouped between these two extremes. The difference being that the two extremes switched positions.

The effect of P_s on percent best possible solution differs greatly from the P_s effect on the percent improvement over the base case. In the percent improvement measure, the effect was a quadratic, monotonically decreasing function of P_s with a minimum near $P_s = 1.0$ for each phase (Figure 30 - 34). In the percent best possible solution, the P_s effect for each phase is different and both differ from the effect of P_s on the percent improvement measure. The percent best possible solution Phase I P_s effect is a monotonically increasing function of P_s. (The value of the multiple-plan system increases faster than the value of the best possible solution as P_s increases, over the entire range of P_s). The Phase II percent best possible solution dips as P_s increases from .2 to .4, and then increases for $P_s = .6$ and higher yet for $P_s = .8$ -- a local minimum occurs somewhere near $P_s = .5$ (Figure 35 - 39). (For low P_s values, the Phase II targeting option dominates. As P_s increases, the value of the Phase II multiple-plan system decreases relative to

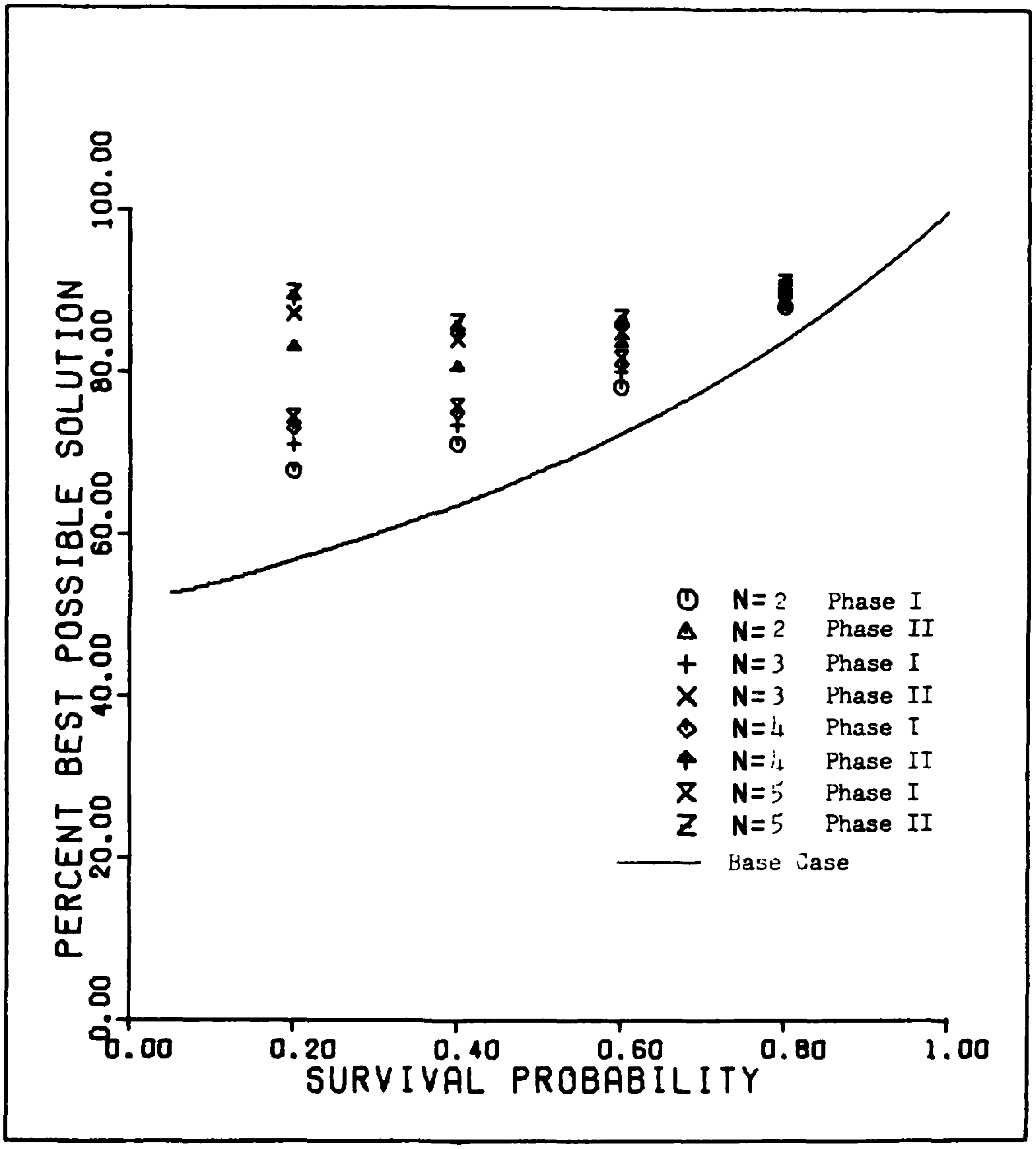

Figure 35. Percent Best Possible Solution -- Integer Distribution

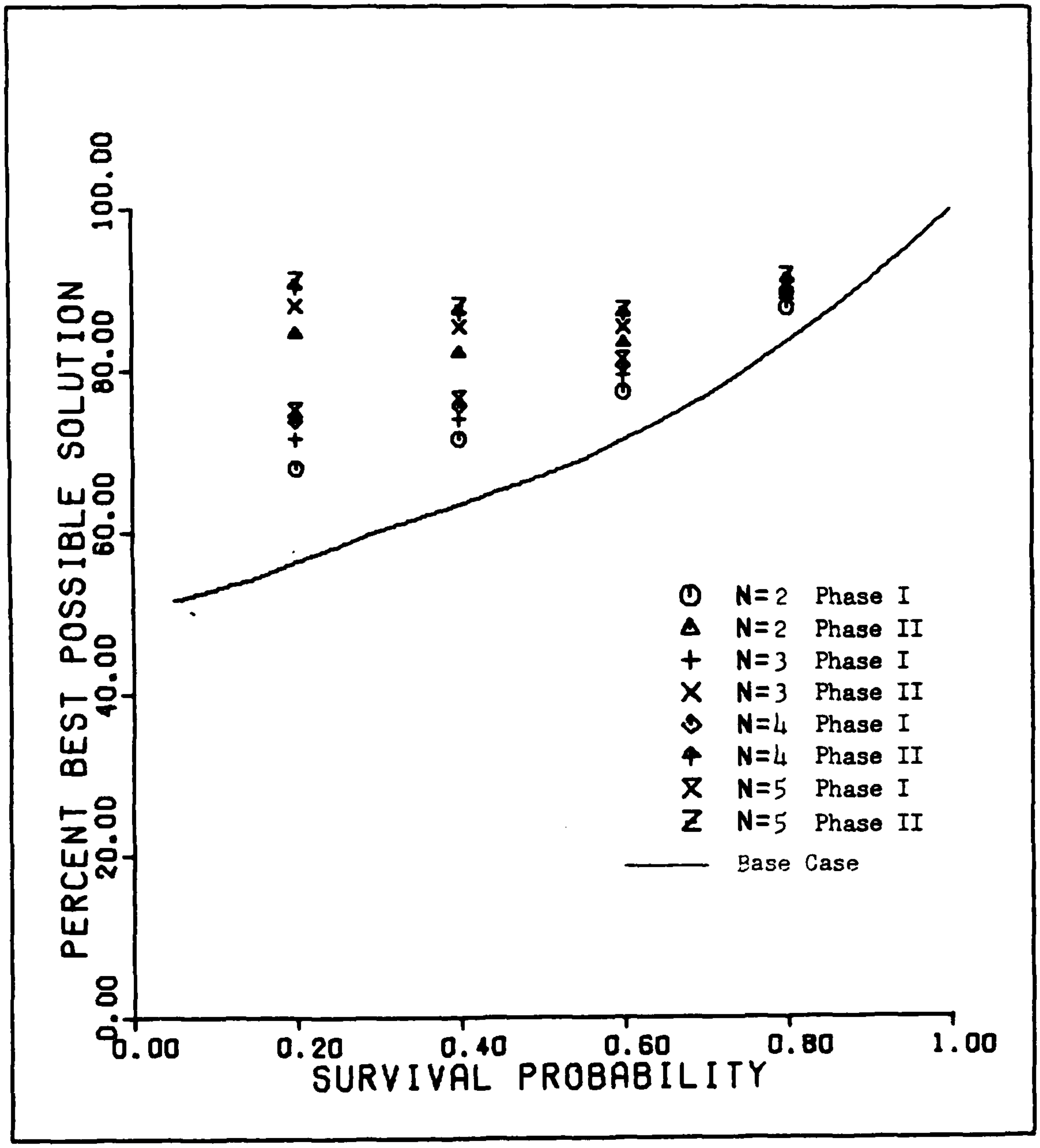

Figure 36. Percent Best Possible Solution -- Uniform Distribution

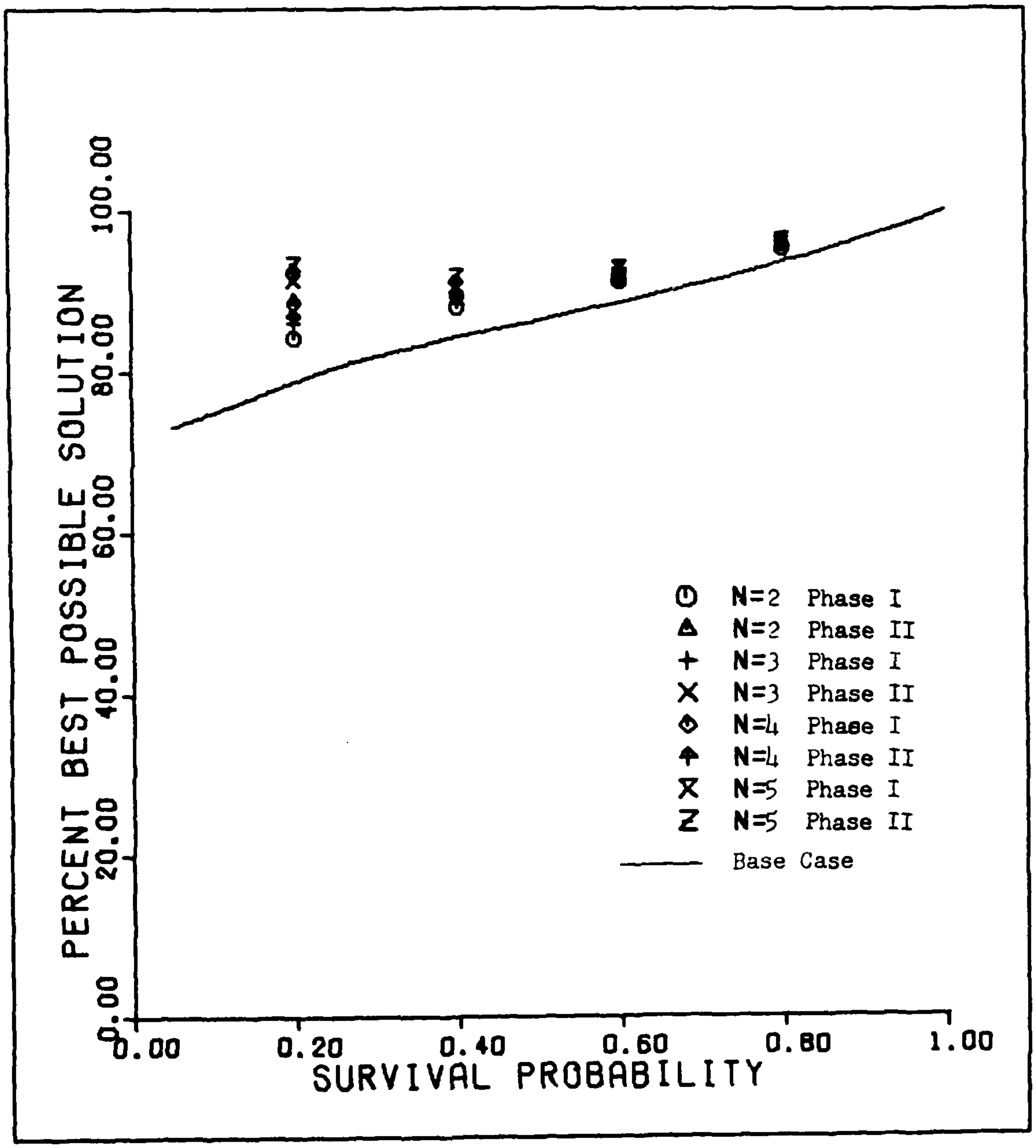

Figure 37. Percent Best Possible Solution -- Low-Variance Normal Distribution

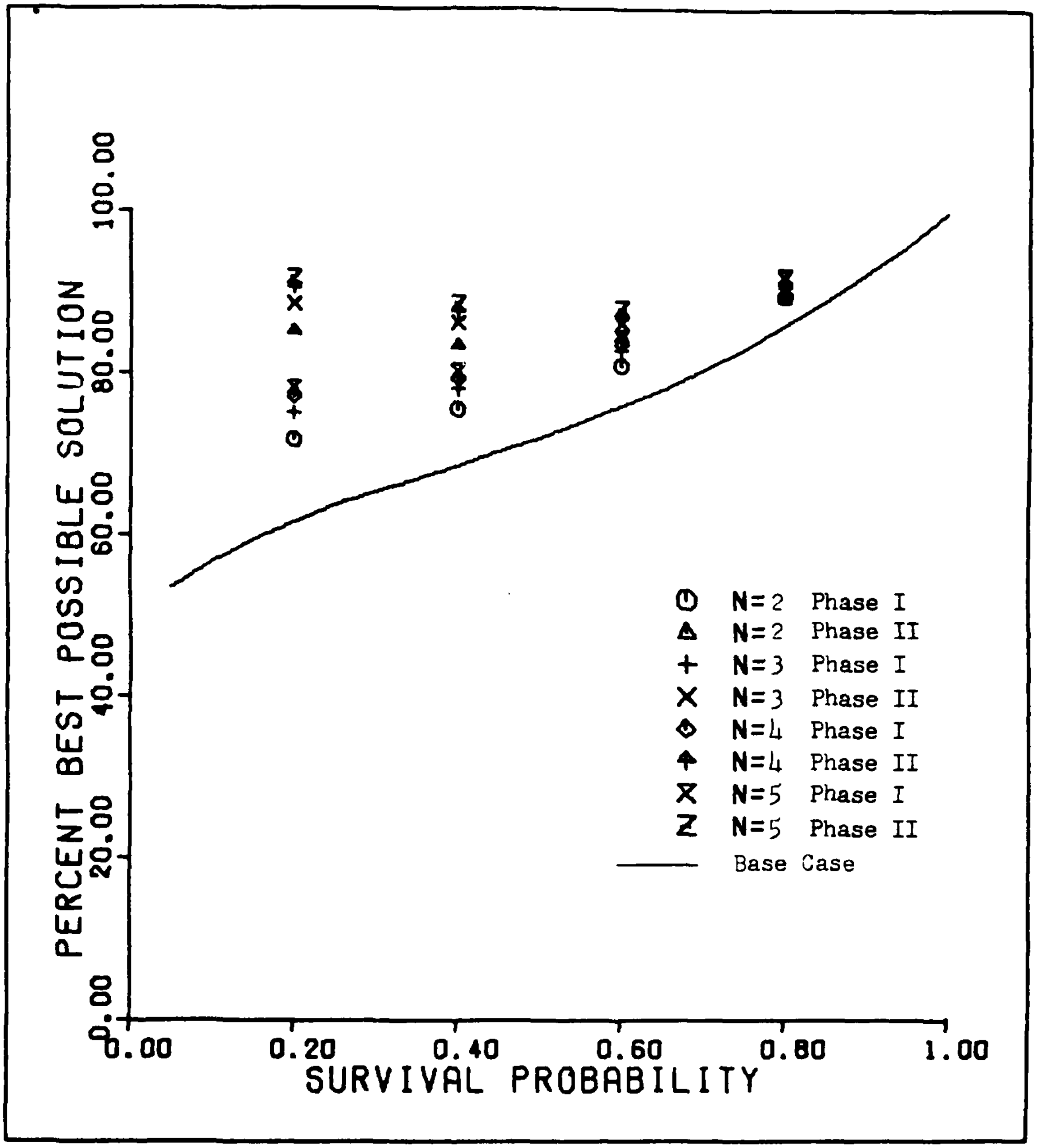

Figure 38. Percent Best Possible Solution -- High-Variance Normal Distribution

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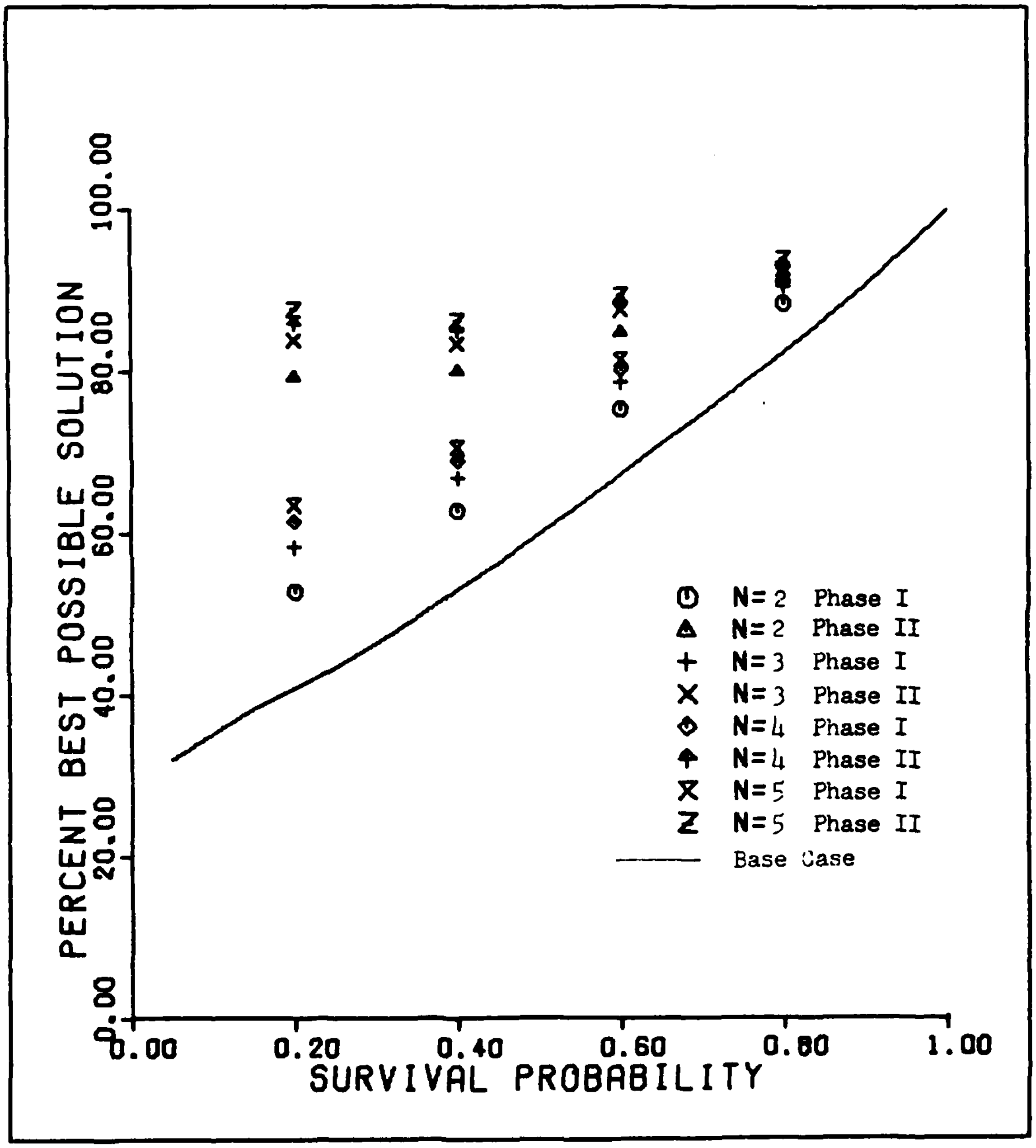

Figure 39. Percent Best Possible Solution -- Exponential Distribution

the value of the best possible solution. For larger P_s values, however, the effect of targeting option decreases. In this P_s range, the value of the Phase II multiple-plan system increases faster than the value of the best possible solution, resulting in an increasing measure. The changeover point occurs somewhere around $P_s = .5$). None of these Phase II deviations, though, are large in magnitude.

According to these results, the MFEV, with only two plans and the Phase II targeting option, would be able to destroy at least 80 percent of the value that the most flexible retargeting system could destroy, for the values of P_s and target value distributions tested. If the MFEV had five plans, around 90 percent could be destroyed. Thus, a large proportion of the value of the total retargeting flexibility is gained by any MFEV system. Whether the remaining ten to twenty percent of the value would be worth the cost of a full inflight targeting system is not considered in this paper.

But, one might ask, how well does the base case do compared to the MFEV? What percent of the value of flexibility is supplied by the MFEV? The value of flexibility is defined here as the difference between the base case curve and one hundred percent of the best possible solution -- in other words, the vertical measure between the line (representing the value of the single-plan system) and the top of the scale in Figures 35 - 39. For example, consider Figure 35, the integer distribution. At $P_s = .4$, the value of flexibility would be about forty percent (100 - 60). Similarly, at $P_s = .8$, the value of flexibility would be about fifteen percent (100 - 85). The Phase I data seems to indicate that the MFEV supplies approximately one-third of the value of flexibility. Phase II seems to do about twice as well or on the order of two-thirds of the value

of flexibility. In other words, fully flexible retargeting could increase the expected value of targets destroyed by half again over the incremental Phase II value.

Percent of Total Value

The percent of total value measure is shown in Figure 40. (Note the horizontal scale is reversed from Taylor's work (Ref 9:2) to remain consistent within this paper). The set of four points from any distribution represent the expected percentage of the total target value destroyed by the best possible solution for each discrete value of P_s. All five distributions are shown. Thus, one hundred percent could only be achieved if all aircraft survived and zero would be the result if no aircraft survived the enemy attack. The line is the lower bound for any distribution of target values. It represents the percent total value for a set of identical targets or the base case -- i.e. no inflight retargeting capability. The quadratic curve is the percent of total value achieved by aircraft attacking a set of n integer-valued targets, as n approaches infinity, and is given by Eq (19). This curve represents an upper bound on any central tendency distribution and could be considered a middle value between extreme distributions.

As one would expect (because of the previous results from the percent improvement over the base case measure), the results from the exponential distribution lie above the results from the other distributions. Similarly, the results from the low-variance normal distribution lie below the other distributions. The other three distributions (integer, uniform and high-variance normal) closely approximate the infinite integer case and are grouped together between the extremes. Thus, in an operational environment, if one had the ability to retarget

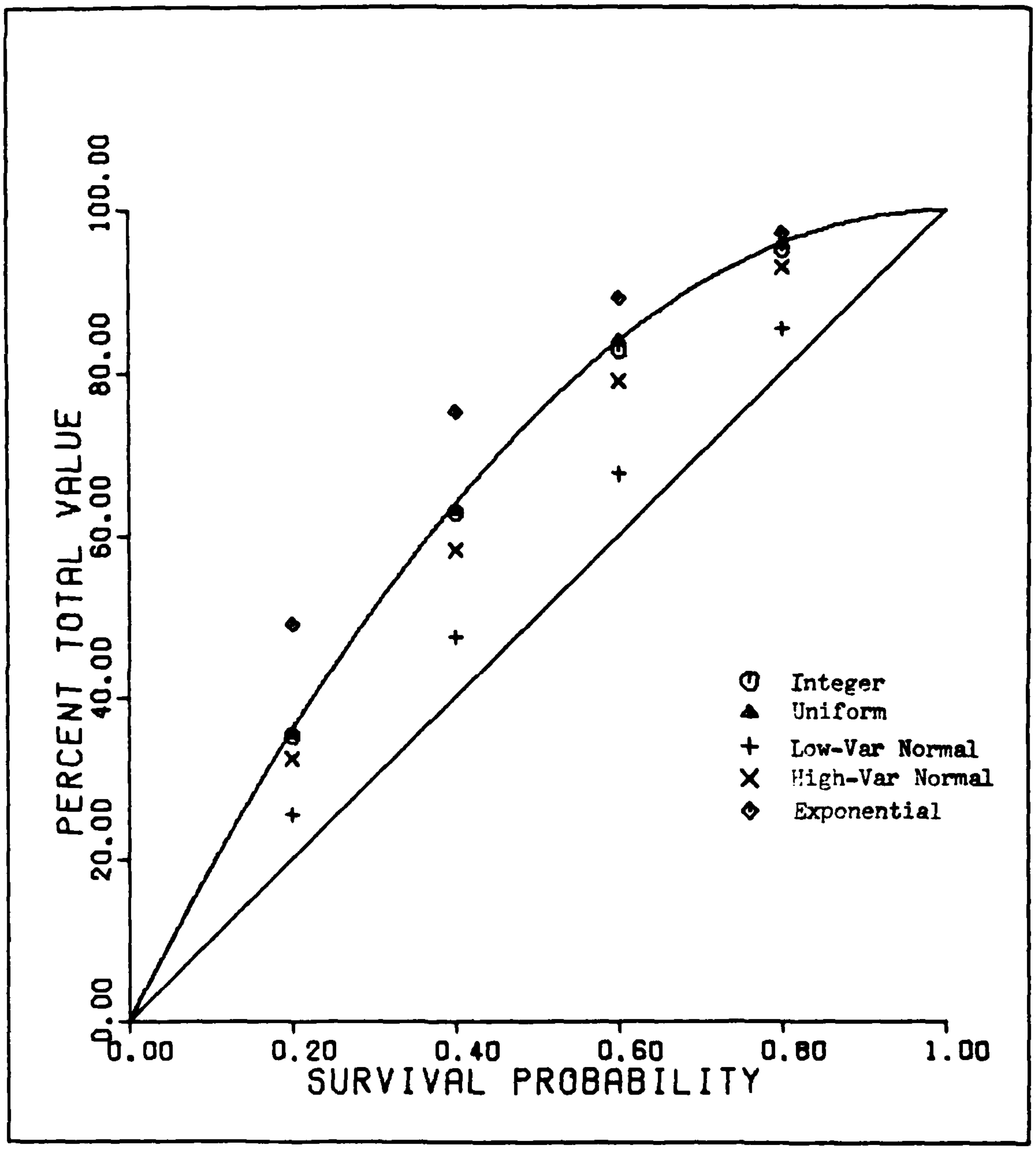

Figure 40. Percent Total Value

in flight, the greatest value would be gained if the set of targets approximated an exponential distribution. Less value would be gained if one were attacking an integer, uniform or high-variance normal distribution and even less value against a low-variance normal. Against identical targets, retargeting is, of course, of no value.

VII Conclusions and Recommendations

This chapter summarizes the results obtained and draws conclusions about the estimated value of the MFEV. Several avenues for future study are suggested.

Conclusions

As was shown in Chapter VI, the value of the MFEV was found to be sensitive to many factors. The impact of these factors on the resulting value will be summarized in this section.

Effect of Targeting Option. For values of P_s in the .8 and above range, targeting option had only a minor effect. This trend seems appropriate since, for P_s values in this range, almost all aircraft would be allocated against different targets anyway, as discussed in Chapters II, III and V. Thus, for $P_s = .8$, assignments under the Phase I and Phase II targeting options are very similar and consequently have similar values.

For smaller values of P_s, however, the value of the Phase II targeting option becomes important. Thus, for smaller values of P_s ($P_s < .6$), the requirement that all targets be assigned in every plan becomes a greater and greater liability. At $P_s = .2$, the Phase II targeting option adds more than twice as much value to the base plan as does the Phase I option. Under the assumption of an exponential target value distribution, the effect is even slightly greater.

Target Value Distribution. The value of the MFEV varies greatly depending upon the assumed distribution of the target values. If there is very little range in the target values (i.e. the targets are nearly homogeneous), it seems reasonable that the benefit gained by trading the ability to kill a lower-valued for the ability to kill a higher-

valued target would be small. Thus, intuition would lead one to believe that the value of the MFEV under an assumption of low-variance normal target values would be smaller than under most other distribution assumptions. Such was found to be the case, as shown in Figure 29.

On the other end of the spectrum lies the exponential distribution. With this distribution, the value of the MFEV was found to be much higher than under any other distribution. This is intuitively appealing because the gap in value between the "average" target and one of the few higher-valued targets would be greater than that in any other distribution tested. Therefore, one might have expected target values following an exponential distribution to yield the highest value. This was exactly what was found.

The other three target value distributions were found to lie between these two extremes. The integer-valued and uniform distributions measured roughly one-half the exponential distribution, given the same conditions. The high-variance normal distribution of target values was slightly lower than these two. The low-variance normal distribution yielded the lowest results, measuring only about one-sixth that of the exponential target values.

<u>Effect of P_s</u>. The probability of aircraft survival was found to be the most influential factor in determining MFEV value. As the P_s decreases, the value of additional plans (to supplement the basic, single-plan assignment) increases. In other words, as fewer and fewer aircraft are expected to survive the initial enemy attack, it becomes more and more important that those aircraft which do survive be allocated against higher-valued targets.

The value of the MFEV was found to be a nonlinear, monotonically

decreasing (within the interval tested) function of the aircraft survival probability. Under every assumption of target value distribution, the value of the MFEV was found to increase by greater and greater amounts as P_s is decreased linearly. Thus, if it is determined that the MFEV is worth having at $P_s = .6$, the value of the MFEV at any $P_s < .6$ is even greater.

Improvement Due to the Number of Preplanned Options. The number of plans in the MFEV system was not found to be a major factor. In all cases, as one would expect, an increase in the number of plans did lead to an increase in the value of the system. The addition of each additional plan, however, only raised the value of the system by about two or three percentage points -- a small improvement when compared with factors such as target value distribution or aircraft survival probability. The number of plans were found to contribute to the value of the system in a linear manner over the range of plans considered. Diminishing marginal returns seemed to be present, but was not modeled due to a multicollinearity problem. Clearly, as the number of plans continued to increase, the marginal return per plan would decrease, eventually approaching zero.

Comparison with Best Possible Solution. For all target value distributions and all tested survival probability levels, a MFEV five-plan system was found to deliver mearly 90 percent of the best possible solution under the Phase II targeting option. This value varied only slightly as a function of P_s, number of plans and target value distribution. The targeting option had a large impact on the value measured, especially for $P_s < .6$.

The value of the best possible solution was found to be somewhat

dependent upon target value distribution. The exponential scored higher and the low-variance normal scored lower than the other target value distributions compared. This relationship was found to be the major source of their rankings in the percent improvement over the base case measure. Thus, for target distributions with very little variety in target values, the MFEV adds little value to a single-plan assignment. Under target value distribution assumptions where a few high-value targets far outweigh the bulk of the targets (such as the exponential) the value of the MFEV can be quite large, especially for small P_s values.

Recommendations for Further Study

This research has attempted to explore a number of facets of the MFEV value question. Many, many additional avenues for study remain. This section will discuss some possible expansions.

Research into the MFEV question could follow at least three separate, although not independent, paths. These could be thought of as the axes of a three-dimensional graph, as shown in Figure 41, with axes labelled Plan Flexibility, System Flexibility and Realism. Each of the directions can be explored to shed light on this seemingly-uncomplicated question of MFEV value.

Plan Flexibility. The plan flexibility axis is characterized by the expansion of targeting flexibility at the plan level. For example, this research was divided into Phase I and Phase II -- Phase II allowed multiple aircraft to be allocated against the same high-value target; while Phase I required distinct targets be selected for each aircraft. This could be considered a one unit movement on the X-axis. This research though, required that each plan have exactly the same set of target assignments -- if three different aircraft were assigned against target

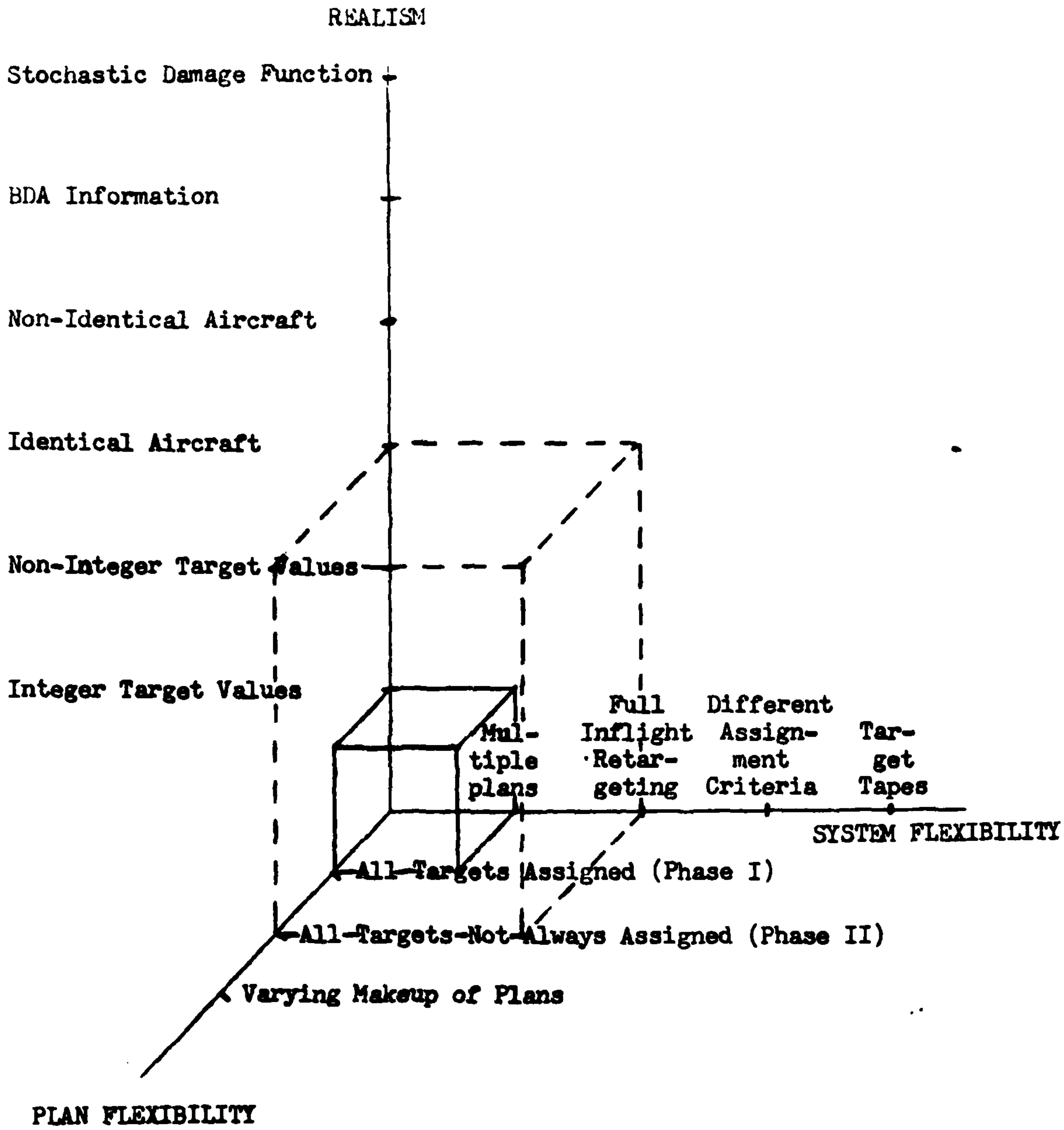

Figure 41. MFEV Research Areas

seven in plan one, plan two (and any other plans) also had three aircraft allocated to target seven. This may not be the best assignment scheme, however, and varying the assignment rationale could be considered a further move out the X-axis.

System Flexibility. Movement along the Y-axis includes all possible variations to the standard assignment procedure which would increase the flexibility of the system as a whole -- changes which would increase the effectiveness of the targeting system by restructuring the manner in which things are done. Increasing the number of preplanned MFEV options is an example of a movement in this direction.

Future research could utilize a different set of criteria to determine the assignment of aircraft to targets over all plans. As was discussed in Chapters II and III, a heuristic algorithm was used in this research to find a near-optimal solution to a problem that is similar to the MFEV assignment problem. These results establish a lower bound on the value of the MFEV. Thus, another solution procedure may very well discover different, higher values.

Assumptions of differing quality of command and control systems based on the MFEV could be the basis for much further research. For example, if one were willing to assume highly effective command and control, even after an enemy first strike, the destruction capability of the bomber fleet could be even further enhanced by allowing full inflight retargeting (subject only to geographical constraints). Thus, surviving aircraft would be able to attack any high-value targets within flying range. Alternatively, one could assume a slightly more restrictive case -- that each aircraft carried a distinct set of target tapes. In event of war, surviving aircraft could attack any target for which they

had a tape and which was within their range. Research under this scenario could include selection of assignment criteria (which tapes to place on board which aircraft) and the value of such a system.

Realism. A third direction future research could expand is the degree to which the modeled assumptions simulate reality. For example, this research assumed the same survival probability (P_s) for all aircraft. Future research could relax this assumption and allow P_s to vary between bases (as a function of distance from the sea, for example -- different missile flight times from enemy submarines) or even individually between aircraft on each base. Relaxing the equal P_s assumption would probably also require a different aircraft to targets assignment scheme.

This research assumed perfect knowledge about the aircraft which survived the initial enemy attack. Another step farther out the information axis could include assumptions about the quality and quantity of Bomb Damage Assessment (BDA) data available in the hours after a return U.S. missile strike but before the bombers reach their targets. The BDA information could consist of reports concerning which targets remained valuable (either were not targeted on the missile strike or were not successfully attacked). This information would be very valuable in retargeting bombers enroute to already destroyed targets, rerouting bombers from low-value to high-value targets, and in deciding which of the preplanned MFEV options should be selected.

Another possible expansion of this research could be in the area of target damage. This research assumed all targets held their value until attacked by a bomber at which time they were totally destroyed with probability 1.0. In future research, a stochastic damage function would be substituted to allow partial destruction and a corresponding

reduced target value. Also, target values which vary as a function of time could be an interesting subject area.

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Second Strike Retargeting

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Value of Retargeting Flexibility

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

The change in the destruction capability of a bomber force resulting from the incorporation of the Multiple Force Employment Variation (MFEV) was estimated under a variety of conditions. The study varied the distribution of target values, the bomber's base escape survival probability, the number of preplanned options, and targeting philosophy. The resulting value of targets destroyed under the MFEV were compared with the value of targets destroyed under a

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single-plan system under identical conditions. The value of the MFEV was also compared with the value of a system with full retargeting capability. Full retargeting capability is defined as the ability to attack the J targets of highest value when exactly J aircraft survive an enemy surprise attack.

Aircraft survival probability, target value distribution and targeting philosophy were all found to have a major effect on the estimated value of the MFEV. (Targeting philosophy , in this paper, refers to the requirement that all aircraft be assigned against distinct targets.)

The number of plans above two was not found to be a strong determinant of MFEV value.

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